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A SPECIAL APPLICATION OF PARTIAL CORRELATION*

By W. L. Crum, *Williamstown, Massachusetts*

It is well known that if two ordered statistical series—for instance, two historical economic series—are subject to rectilinear trends, their correlation coefficient is influenced by that fact. Since it is ordinarily desired to determine the degree of correlation which is independent of the normal trends, it becomes necessary to separate in each series the variation due to trend from the miscellaneous variation. A common method† of accomplishing this consists in determining the line of trend and correcting the items of the given series by corresponding amounts. In other words, the plan is to study the deviations of the items from the line of trend rather than the horizontal. The object of this paper is to interpret this process in terms of the partial correlation coefficient and to call attention to a corresponding direct method of eliminating the secular trend.

I. STANDARD DEVIATION

Suppose that a single series of N items, x_i, is ordered relative to a particular variable which, although it need not be the time, will be represented by t; and assume that the series has a trend which is substantially rectilinear. It is sought to find this line of trend and to determine the standard deviation of the x_i relative to it. From among the methods of locating the line, we adopt that of least squares.

Let the equation of the line be

$$x = bt + a$$

and let ξ_i be the residual obtained by diminishing x_i by the above value

* Read before the American Mathematical Society on September 9, 1921, under the title "The significance of the partial correlation coefficient in the comparison of ordered statistical series possessing rectilinear trends."

† Persons, Warren M., "An Index of General Business Conditions," in particular Part II ("Method used"), in *Review of Economic Statistics*, April, 1919.

of the ordinate x for t equal to t_i. Then the function which is to be made minimum is

$$\begin{aligned}\Sigma\xi^2{}_i &= \Sigma(x_i - bt_i - a)^2 \\ &= \Sigma(x_i - a)^2 - 2b\Sigma t_i(x_i - a) + b^2\Sigma t^2{}_i\end{aligned}$$

and the conditions furnishing a minimum are

$$a = X - bT, \text{ where } X = \frac{1}{N}\Sigma x_i, \text{ and } T = \frac{1}{N}\Sigma t_i \tag{1}$$

together with:

$$b = \frac{\Sigma t_i(x_i - a)}{\Sigma t^2{}_i}. \tag{2}$$

The first of these conditions expresses the fact that the line passes through the point having as coördinates the arithmetic means of the t_i and x_i, and the second gives the slope of the line of best fit.

The corresponding minimum value of the function is

$$\Sigma\xi^2{}_i = \Sigma(x_i - a)^2 - b^2\Sigma t^2{}_i = \Sigma(x_i - X)^2 - b^2(\Sigma t^2{}_i - NT^2)$$

giving

$$\sigma^2{}_\xi = \sigma^2{}_{x.t} = \sigma^2{}_x - b^2\sigma^2{}_t \tag{3}$$

where σ_x is the ordinary standard deviation of the x_i, and σ_ξ is the standard deviation of the residuals. In other words, σ_x is the standard deviation of order zero, and σ_ξ — or, $\sigma_{x.t}$ — is that of the first order.*

It is evident that $b^2\sigma^2{}_t$ can never exceed $\sigma^2{}_x$, and can equal it only in case the original distribution is precisely rectilinear; and, moreover, $b^2\sigma^2{}_t$ can never be negative, and is zero only if the trend is horizontal. Hence, $\sigma_{x.t}$ is zero only if the actual distribution is exactly rectilinear, and has its maximum value of σ_x only in case the trend is horizontal.

It is suggested that for certain purposes the coefficient $\sigma_{x.t}$ is a better measure of fluctuation than σ_x. Whereas σ_x is ordinarily used to measure dispersion, $\sigma_{x.t}$ indicates the degree of divergence of the items from the line of normal trend. The difference between the squares of the two, $b^2\sigma^2{}_t$, is the square of the standard deviation of the ordinates of the line of trend. In calculating $\sigma_{x.t}$ no correction for trend need actually be made in the given items: the value is obtained directly by use of equation (3), and the work is particularly easy if the t_i are successive integers.

Since the coefficient of correlation† between x and t is defined by

$$r_{xt} = b\frac{\sigma_t}{\sigma_x} \tag{4}$$

* Yule, G. U., *An Introduction to the Theory of Statistics*, chap. xii, sec. 6; 5th ed., London; Charles Griffin & Co. 1919.

† *Ibid.*, chap. ix, sec. 10.

equation (3) may be written in the form *

$$\sigma^2_{x.t}=\sigma^2_x(1-r^2_{xt}). \tag{5}$$

II. CORRELATION

We address ourselves next to the examination of the correlation between one ordered series, x_i, having rectilinear trend defined by the parameters a_x and b_x, and a second, y_i, with trend defined by a_y and b_y. Let it be supposed at the start that the correlation is not subject to any lag.

The equations of the two lines of trend are

$$x=b_xt+a_x,\ y=b_yt+a_y$$

and, if ξ_i and η_i are the residuals obtained by diminishing the ith items of the two series by the ith ordinates of the respective lines of trend,

$$\xi_i=x_i-b_xt_i-a_x,\ \eta_i=y_i-b_yt_i-a_y.$$

It was remarked at the beginning that the correlation sought is that which exists between these residuals, namely, $r_{\xi\eta}$. Its calculation involves the product sum Np, where

$$\begin{aligned}Np&=\Sigma\xi_i\eta_i\\&=\Sigma(x_i-b_xt_i-a_x)(y_i-b_yt_i-a_y)\\&=\Sigma(x_i-a_x)(y_i-a_y)-b_xb_y\Sigma t^2_i\\&=\Sigma(x_i-X)(y_i-Y)-b_xb_y(\Sigma t^2_i-NT^2)\\&=(r_{xy}-r_{xt}r_{yt})N\sigma_x\sigma_y\end{aligned}$$

by use of (1), (2), (4). Furthermore, using (4) and (5) with this result,

$$r_{\xi\eta}=\frac{r_{xy}-r_{xt}r_{yt}}{\sqrt{1-r^2_{xt}}\ \sqrt{1-r^2_{yt}}}=r_{xy.t}. \tag{6}$$

We find, therefore, as might have been foreseen, that the correlation between ξ_i and η_i, as ordinarily calculated by "correcting" the variates x_i and y_i, is merely the partial correlation† of x_i and y_i, independent of t. Thus, in calculating the correlation between ξ_i and η_i, we are relieved of the necessity of correcting for trend: the arithmetic involved in computing the residuals ξ_i and η_i is eliminated entirely. The simpler process is to use formula (6) directly: the coefficients r_{xt} and r_{yt} must be evaluated anyway in order to locate the lines of trend, and the labor of determining r_{xy} is no greater than that of finding $r_{\xi\eta}$.

* *Ibid.*, chap. ix, sec. 14; and chap. xii, sec. 3.
† *Ibid.*, chap. xii, sec. 13.

It is to be noted that this method applies only for the elimination of rectilinear trends. If it is desired to correct also for so-called seasonal variation before examining correlation, the computation of the partial correlation coefficient will not automatically effect that correction; but even in this case it will eliminate the rectilinear factors.

III. EFFECT OF LAG

It remains to inquire into the influence of lag upon the conclusion of Section II. Suppose that y lags behind x by an amount L: that t'_i, the value of t associated with y_i, is L less than t_i, the value of t associated with x_i. The residuals are, on this assumption,

$$\xi_i = x_i - b_x t_i - a_x, \quad \eta_i = y_i - b_y t'_i - a_y$$

and the product sum takes the form:

$$\begin{aligned} Np = \Sigma \xi_i \eta_i &= \Sigma (x_i - b_x t_i - a_x)(y_i - b_y t'_i - a_y) \\ &= \Sigma (x_i - a_x)(y_i - a_y) - b_x b_y \Sigma t_i t'_i \\ &= \Sigma (x_i - X)(y_i - Y) - b_x b_y (\Sigma t_i t'_i - NTT') \\ &= \Sigma (x_i - X)(y_i - Y) - b_x b_y \Sigma (t_i - T)(t'_i - T') \\ &= (r_{xy} - r_{xt} r_{yt'} r_{tt'}) N \sigma_x \sigma_y \end{aligned}$$

giving

$$r_{\xi\eta} = \frac{r_{xy} - r_{xt} r_{yt'} r_{tt'}}{\sqrt{1 - r^2_{xt}}\ \sqrt{1 - r^2_{yt'}}}. \tag{7}$$

It is evident that the value given by (7) is not exactly the partial correlation coefficient, but differs from it by the multiplier $r_{tt'}$ in the second term of the numerator. Nevertheless, the calculation of the lagging correlation between ξ_i and η_i is facilitated by the use of (7) rather than the laborious method of correcting the original items. As the t_i and t'_i differ ordinarily by a constant lag, $r_{tt'}$ will generally be unity in the cases arising in practice. In fact, we have

$$t'_i = t_i - L$$

and, if the lag is constant, the arithmetic means T, T' will be related by

$$T' = T - L.$$

Hence quantities of the type $t'_i - T'$ reduce to $t_i - T$, and the numerator and denominator of $r_{tt'}$ become identical: the coefficient of correlation between t and t' reduces to unity, as might have been foreseen, since the relation between the two t series is perfect when L is constant.

Therefore formula (6) and the conclusion of Section II still hold when there is lag, provided the lag is constant. This will cover all the simpler cases arising in practice. Should it be desirable, however, to compare two series with variable lag, formula (7) can be used; and the calculation of the coefficient $r_{tt'}$ will not greatly complicate the work.

A PRICE INDEX OF OIL STOCKS

By Joseph E. Pogue, *New York*

For a number of years the writer has been conducting a continuing economic study of the petroleum industry, based partly upon the relatively ample statistics with which this field is favored and partly upon the technology involved in the production, refining, and utilization of petroleum.* In order to ascertain whether the course of petroleum shares on the stock market is influenced by economic conditions peculiar to petroleum or is wholly a function of the forces common to the movement of industrial shares in general, a price index of oil stocks has been prepared and will be maintained. A brief description of the method employed and the results attained is herewith presented.

The preparation of a practical stock index involves greater difficulties than many other types of indices, owing to the rapidity and the degree of scatter characteristic of the individual quotations, and the tendency of some of the items to die away and thus "load" the index with inactive components. The conventional type of index as calculated by financial journals and consisting merely of an arithmetic mean of a selected number of quotations is a crude expedient, as has been critically shown by Wesley C. Mitchell.† The main trouble with this type of index is that it is dominated by the more vigorous of its components and is, therefore, not a representative index.

Obviously, if the more vigorous and more costly stocks are not to dominate in an index, the items must be weighted. The weights may be assigned directly to the items or introduced indirectly by converting the quotations into percentages of a fixed base. The latter method was preferred because it affords a double comparison and lends itself to convenient treatment.

In the preparation of the oil index twenty petroleum stocks were selected: five Standard Oil shares; 5 shares of representative companies listed on the New York Stock Exchange; 5 shares of companies listed on the New York Curb; and 5 shares of companies operating primarily in foreign countries. A monthly average price was calculated by averaging the closing quotation on Friday of each week. The monthly averages so obtained were then converted into index numbers representing for each item the percentage of its average price in 1919.

* The writer believes that a combination of statistics and technology opens up a very fertile field for research in industrial economics.

† "A Critique of Index Numbers of the Prices of Stocks." *Journ. Polit. Econ.*, July, 1916, pp. 625–93.

The next step was the calculation of a representative average of the twenty index numbers for each month. An arithmetic average offers the objection that the stocks which decline most rapidly in price have a decreasingly important effect upon the average. This difficulty is largely overcome by use of the geometric mean which accentuates the weight of the smaller index numbers. The median was also determined.

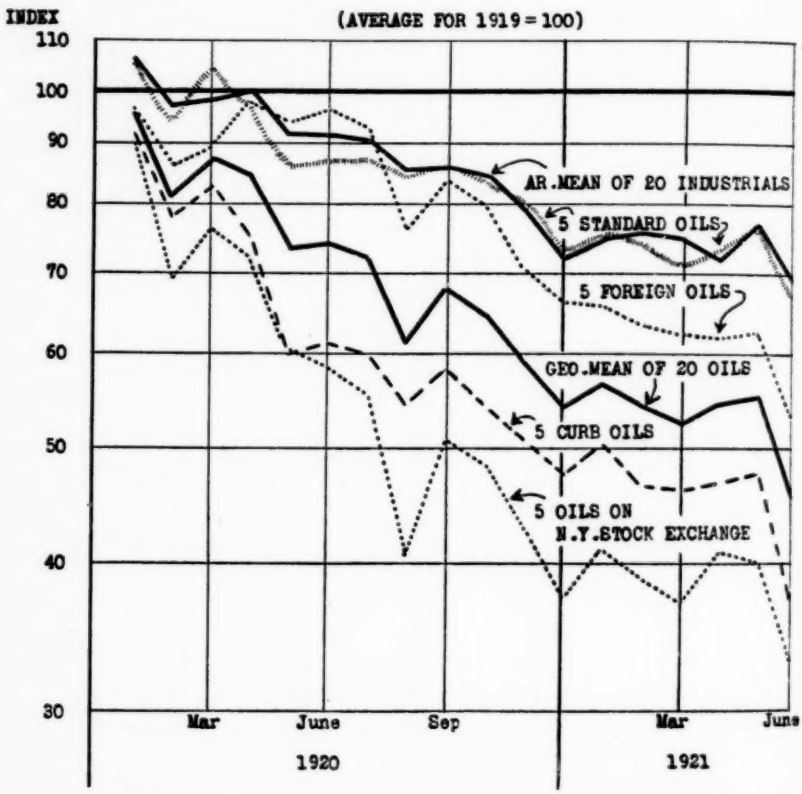


FIGURE 1.—Comparison of the geometric mean, arithmetic mean, and median of 20 oil stocks by months, 1920–21.

The three composites so obtained—geometric mean, arithmetic mean, and median—are given on a semi-logarithmic scale in Figure 1 by months for 1920–21. It will be observed that the three curves show closely concordant fluctuations, but the geometric mean falls below the arithmetic mean, and both show a greater decline than the median. The geometric mean indicates sharper month-to-month fluctuations than does the arithmetic average. The median seems

somewhat erratic in months of abrupt change such as June. The data on which Figure 1 is based are given in Table I.

The geometric mean of the 20 oils and of each of the four component groups are plotted on a semi-logarithmic scale in Figure 2. The arithmetic mean of 20 industrials, as calculated by Dow, Jones, and Co. and published in Bradstreet's, is also entered upon the chart for purposes of comparison after the figures have been reduced to percentages of their average in 1919. It is to be noted that the curve for 20 oils moves in conformance with the curve for 20 industrials, but the month-to-month fluctuations for oils are more accentuated and the trend over the period shown is more sharply downward than for industrials in general. The reason for this is that the oil shares are more "speculative" than the run of items entering in the average industrials.

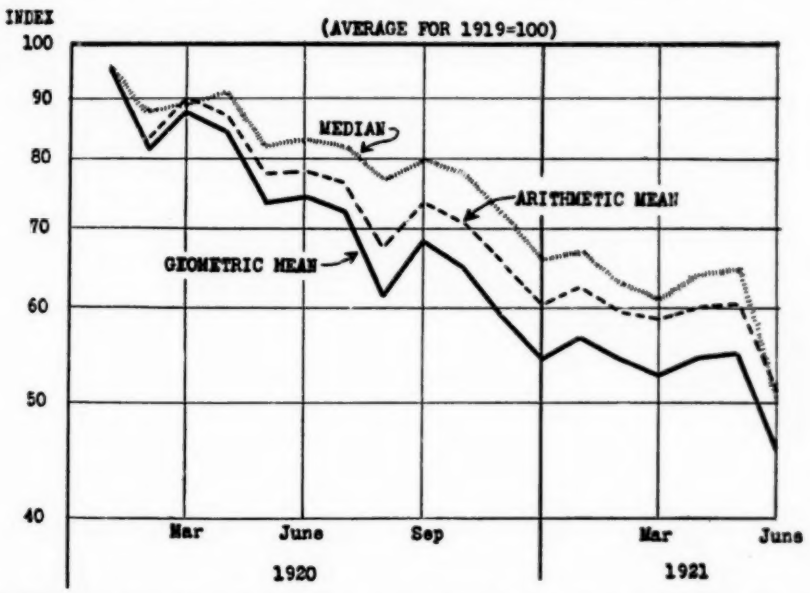


FIGURE 2.—Price of 20 oil stocks by months, 1920–21, together with prices of four component groups, compared with the price of 20 industrial stocks.

Turning next to the components of the oil average, while we recognize that five items form a rather meager sample for each group, we nevertheless observe a certain consistency. The Standard oils, representing the most stable class of oil stocks, conform very closely to the course of the 20 industrials. The foreign oils follow to some degree the course of 20 industrials during 1920, but depart downward rather sharply in 1921. The Curb oils and the New York Stock Exchange

oils are next in order; the latter show the more decided month-to-month fluctuations and the greater ultimate decline from the 1919 average. Whether these relationships will be maintained will be disclosed as the indices are calculated over the months ahead.

TABLE I

PRICES OF OIL SHARES BY MONTHS, 1920–21, COMPARED WITH PRICES OF 20 INDUSTRIALS, IN PERCENTAGES OF THE FIGURES IN 1919

	Geometric mean of					Arith. mean	Median	Arith. mean
	5 Standard oils	5 Foreign oils	5 Curb oils	5 oils on N. Y. Stock Exchange	20 oils	20 oils	20 oils	20 industrials
1919:								
Year.....	100	100	100	100	100	100	100	100
1920:								
Jan.....	105.3	96.5	92.0	90.5	95.9	96.6	96	106
Feb.....	94.6	86.6	78.3	69.1	81.6	83.0	88	97.0
Mar.....	103.4	89.8	83.2	76.6	87.7	89.6	89	98.2
Apr.....	96.4	97.8	75.2	72.5	84.6	87.1	91	99.8
May.....	86.1	94.0	60.6	60.8	73.9	77.7	82	91.3
June.....	87.4	96.1	61.4	58.8	74.2	78.1	83	92.0
July.....	87.1	93.0	60.0	55.8	72.2	76.1	82	91.0
Aug.....	84.9	76.9	54.6	40.9	61.7	67.5	77	85.5
Sept.....	86.6	84.0	58.6	50.9	68.3	73.6	80	86.5
Oct.....	83.7	80.0	54.5	48.6	64.9	70.9	78	85.0
Nov.....	80.2	70.9	51.0	42.8	59.3	65.7	72	79.5
Dec.....	73.5	66.6	47.6	37.4	54.3	61.1	66	72.3
1921:								
Jan.....	75.8	66.0	50.5	41.1	56.8	62.3	67	75.0
Feb.....	74.2	63.4	46.7	38.8	54.3	59.7	63	76.0
Mar.....	71.7	62.6	46.2	37.1	52.7	58.8	61	75.2
Apr.....	73.9	62.1	46.9	41.0	54.5	60.0	64	72.1
May.....	76.4	62.8	47.7	40.1	55.0	60.5	65	77.0
June.....	67.1	53.0	37.1	33.0	45.7	51.0	50	69.3

In order to compare the price trend of oil shares with the course of commodity prices in the petroleum industry, index numbers representing the weighted average price of crude petroleum and of petroleum products are compared with the index numbers of 20 oil shares in Table II. The weighted average for crude petroleum represents five grades, and the weighted average for petroleum products is a weighted composite of individual averages for gasoline, kerosene, fuel oil, and lubricating oils.* The tabulation shows that the price of oil shares over the period shown declined independently of the trend of commodity prices in the petroleum industry, following the course of the

* Corresponding monthly index numbers for the entire period, 1913–21, will be found in Pogue, *The Economics of Petroleum*, New York, 1921, from which the commodity numbers shown above were recalculated.

stock market. This, of course, is normal, although it is interesting to note that the sharp advance in the price of crude petroleum and its products that characterized the first quarter of 1920 was without apparent effect upon the price of oil shares in the stock market.

TABLE II

COMPARISON OF THE AVERAGE PRICES OF CRUDE PETROLEUM, PETROLEUM PRODUCTS, AND 20 OIL SHARES, IN PERCENTAGES OF THE FIGURES IN 1919

	Crude petroleum	Petroleum products	20 oil shares
1919:			
Year	100	100	100
1920:			
Jan.	127	119	95.9
Feb.	131	127	81.6
Mar.	152	140	87.7
Apr.	156	143	84.6
May	158	147	73.9
June	158	152	74.2
July	159	148	72.2
Aug.	159	149	61.7
Sept.	159	149	68.3
Oct.	159	144	64.9
Nov.	158	139	59.3
Dec.	155	135	54.3
1921:			
Jan.	146	125	56.8
Feb.	97	111	54.3
Mar.	88	103	52.7
Apr.	89	101	54.5
May	80	89	55.0
June	64	86	45.7

In conclusion, the geometric mean of fixed-base relatives is employed as a satisfactory type of index number for the price of oil shares, and it is shown that the price course of oil shares in the stock market during 1920–21 followed the price trend of industrial shares in general, though registering sharper fluctuations and greater declines than the latter. Also, the price of oil shares was uninfluenced to any substantial degree by the sharp advance in the prices of crude petroleum and its products in early 1920.

ON THE APPORTIONMENT OF REPRESENTATIVES

By F. W. Owens, *Cornell University*

The problem of apportionment of representatives has its basis in the concept of the word *representative*, and whom or what he represents. For the purpose of definiteness, the problem will be restricted to the question of apportionment of representatives in the House of Representatives of the United States, except where specified. However, much of the discussion is applicable to any similar apportionment.

As applied to the United States the problem has two interpretations, one taking the question as an abstract one, free from any limitations, constitutional or otherwise, except such as may be deemed just or appropriate on their own merits, and the second treating the problem subject to the limitations imposed by the Constitution. As the latter is the interpretation used in this discussion, it is necessary to note briefly the nature of these restrictions or instructions.

The provisions of the Constitution bearing on the subject are as follows:

> Art. I. Sec. II, 3. Representatives and direct taxes shall be apportioned among the several states which may be included within this Union according to their respective numbers, which shall be determined by adding to the whole number of free persons, including those bound to service for a term of years, and excluding Indians not taxed, three fifths of all other persons. The actual enumeration shall be made within three years after the first meeting of the Congress of the United States, and within every subsequent term of ten years, in such manner as they shall by law direct. The number of representatives shall not exceed one for every thirty thousand, but each State shall have at least one Representative; and until such enumeration shall be made . . .
>
> Sec. IX, 4. No capitation or other direct tax shall be laid, unless in proportion to the census or enumeration hereinbefore directed to be taken.
>
> Amendment. Art. XIV. 2. Representatives shall be apportioned among the several states according to their respective numbers, counting the whole number of persons in each state, excluding Indians not taxed.

Aside from internal contradictions, any method of apportionment must be judged by the completeness with which it satisfies the above provisions. It is at once obvious that if "according to their respective numbers" is accepted as meaning *proportional to*, as it has always been accepted, an apportionment can rarely be made which exactly satisfies the conditions. Various approximations to such an apportionment have been used or suggested. As to the nearness of such approximations, there is no agreed standard of measurement. The language of

the apportionment provision being such that exact compliance is impossible, Congress has proceeded to apportion according to what seemed to its members the nearest approach to the intent of the authors of the Constitution. These interpretations themselves have varied from time to time, but the authority of Congress to make such interpretations has never been questioned, probably on account of the provision which makes each House the sole judge of the qualifications of its members.

The first interpretation of Congress was that a number of inhabitants should be chosen arbitrarily and the population of each state should be divided by this number, and the quotient should give the number of representatives to be assigned to the state, *all fractions being neglected.* The method did not cause any disagreement of consequence until after the census of 1830, when a divisor was used which gave to Massachusetts a fraction of 0.99, and Daniel Webster argued that the phrase "according to their respective numbers" should be interpreted as "according to their respective numbers as near as may be," and that a fraction of more than one-half should entitle the state to an extra representative. His contention was not heeded in the apportionment of 1830, but in all later apportionments it has been used as a guiding principle, with minor variations. In 1840 and again in 1910, the method of an assumed ratio, as above, was used, and each state having a fraction greater than one-half was given the extra representative. From 1850 to 1900, a slightly different procedure, known as the Vinton method, was used. It differed from the above method in that first a total size for the House was chosen and the population of the country divided by the assumed size of the House, and this number (the average size of a congressional district) was then used as a divisor of the population of each state, giving for each state a whole number plus a fraction. Representatives were then assigned to the states for all of the whole numbers, and then to the states having the largest fractions, in order of size, until the total number of representatives was brought up to the assumed size of the House. One difficulty found with the method was that sometimes major fractions would not be assigned a representative by this method; in such cases Congress *arbitrarily* assigned extra representatives for such fractions, which varied the method. A lesser difficulty was that states with minor fractions sometimes received an extra representative by the method. This, however, did not seem to meet with any great objection in Congress. Another objection to the method, since known as the Alabama Paradox, is that an increase in the total size of the House may cause a decrease in the number of representatives assigned to some state when this method is

used. When Congress found this to occur, a change in the total size of the House was made such that for the new total the paradox did not arise.

The method now known as the method of major fractions was devised by Professor W. F. Willcox, of Cornell University, to correct what appeared to be weaknesses in the Vinton method while preserving the general principle of allowing an extra representative for each major fraction. This was done by going back to the choice of an independent ratio, or size of congressional district, the method differing from that of 1840 only in that a method of choosing the ratio was shown which would yield any desired total size of House and keep the major fraction principle. The method is also free from the Alabama paradox. One great advantage possessed by this method, from the practical standpoint, was that it led to a priority table which showed just the order in which representatives should be assigned to the different states, up to any desired total, provided the method were used. None of the previous methods did this.

Another suggested method, which has never been used by Congress, was devised by Dr. Joseph Hill, of the Bureau of the Census, and called the method of Alternate Ratios. He contended that the extra representative should be given to a state for which the fraction exceeded that for the geometric mean between the preceding and the following whole numbers. If the true ratio (total population divided by total size of House) is used this might involve the Alabama paradox, but an *arbitrary* ratio can always be so chosen that this will not be the case. In devising this method Dr. Hill took the position that there is nothing in the problem analogous to the distribution of money. We might try to analyze this question. The Constitution puts both into the same sentence, saying "Direct taxes and representatives shall be apportioned among the several states according to their respective numbers." This would seem to indicate that the authors of the Constitution considered the problems involved to be equivalent. If one considers them equivalent, he will naturally say that the problem of apportioning any number of representatives, say 435, is equivalent to the problem of dividing a sum of money, say $435,000,000, among the states "according to their respective numbers" subject to the restrictions that the distribution must occur in lump sums of $1,000,000, and so that each state must have at least one share. In order that the share of each inhabitant should be as nearly as possible the same, the method of distribution needs be such that the mathematical expectation* of the resident of each state will be the same as that of the resident of any other state. This will inevitably give rise to some form

* The mathematical expectation is defined as the product of the probability of an event by the reward obtained in case of the event.

of the major fraction method, as may be seen as follows: If r is thought of as an independent variable, the unknown population of a state, divided by this r is equally likely to give any remainder from O to $r-1$. If the arithmetic mean is taken as the point of distinction as to whether the extra representative or the extra $1,000,000 is to be assigned, the cases in which the extra assignment is made will just balance those in which it is not made, and the "expectation" of every resident will be the correct number. If the geometric mean were used, however, these cases would not balance, as the geometric mean is less than the arithmetic mean, and the expectation of the resident in the small state would exceed that of the resident of the larger state, as the difference between these means grows less as the numbers become larger. Hence, if the analogy between the distribution of money and the apportionment of representatives is granted, from the standpoint of mathematical expectation, a method based on arithmetic mean should be used.

The Method of Equal Proportions devised by Professor E. V. Huntington* leads to exactly the same apportionment as that of Dr. Hill, provided that one uses the proper value of r (not the true value) in applying Dr. Hill's process, but his reasoning is considerably different. He uses the four equations

$$\frac{N_i}{P_i}=\frac{N_j}{P_j};\quad \frac{P_i}{N_i}=\frac{P_j}{N_j};\quad \frac{N_i}{N_j}=\frac{P_i}{P_j};\quad \frac{N_j}{N_i}=\frac{P_j}{P_i}$$

as applied to any pair of states. In order to treat these four equations in the same way, he introduces a measure of variation from equality which shows the same variation from equality in all of the four cases. This measure is variously taken as the difference divided by the larger, or by the smaller, or by some average of the two numbers, which quotient gives a sort of *relative* difference. By this criterion and the further postulate that in a satisfactory apportionment there should be no pair of states which is capable of being improved by a transfer of representatives within the pair, such *improvement* being measured by his type of relative error, he is led to the priority table

$$\begin{array}{llll}
\dfrac{P_1}{\sqrt{1\cdot 2}}, & \dfrac{P_1}{\sqrt{2\cdot 3}}, & \dfrac{P_1}{\sqrt{3\cdot 4}}, & \cdots \\
\dfrac{P_2}{\sqrt{1\cdot 2}}, & \dfrac{P_2}{\sqrt{2\cdot 3}}, & \dfrac{P_3}{\sqrt{3\cdot 4}}, & \cdots \\
\cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot &
\end{array}$$

* For details concerning this method see QUART. PUB., Sept. 1921, pp. 859–70.

and he assigns one representative to each state and then the remaining $N-48$ representatives to the states having the $N-48$ largest entries in the table, in the order of their size. As the denominators in the above table are the geometric means between the successive integers, the name "method of the geometric mean" would be appropriate, and the relationship with the method of Dr. Hill is evident. For the rest of this paper this apportionment, without regard to its source, will be called the method H.

The application of the method of least squares was made to this problem for the first time by the present writer.* For a given number of representatives, N, $\frac{N}{P}$ is regarded as the average interest of a resident in a representative. Similarly if N_i is the number of representatives assigned to a state S_i, $\frac{N_i}{P_i}$ will be the actual interest in a representative for each resident in the state S_i. The difference between these numbers represents the injustice done the individual resident of the state, or the error in his assignment. As a measure of the total injustice under a given apportionment of N representatives we may take

$$Q = \sum_{i=1}^{i=48} P_i \left(\frac{N}{P} - \frac{N_i}{P_i} \right)^2 .$$

Part of the reasons for squaring the errors lie in the mathematical theory of probability and will not be discussed here. The method has a long standing in the mathematical world, both on account of its theoretical basis in probability and on account of the many useful purposes which it has served. Two strong reasons for the squaring should be at once obvious: all of the terms are thereby made positive, and the effect of over-representing one individual will not then cancel the effect of under-representing another. Also the introduction of large errors is discouraged as the square increases the relative effect of a large error. Without this squaring, even if absolute values were taken, if two individuals were under-represented, to further under-represent one and give the same amount to the other but still leaving him under-represented, would not change the total, and we wish to have every error as well as the total small.

With this basis, the problem is to choose such a distribution of the total number, N, of representatives as will make the total injustice, Q, done to all the residents of the country by the variations in their inter-

* In a paper presented to the American Mathematical Society, February 26, 1921.

est in a representative, as small as possible. That is $N_1, N_2, \ldots N_{48}$ are to be chosen so that

$$Q=\sum_{i=1}^{i=48} P_i\left(\frac{N}{P}-\frac{N_i}{P_i}\right)^2$$

is to be as small as possible.

But
$$\begin{aligned} Q&=\Sigma P_i\left(\frac{N^2_i}{P^2_i}-\frac{2NN_i}{PP_i}+\frac{N^2}{P^2}\right)\\ &=\Sigma\frac{N^2_i}{P_i}-2\Sigma\frac{N}{P}N_i+\frac{N^2}{P^2}\Sigma P_i\\ &=\Sigma\frac{N^2_i}{P_i}-2\frac{N}{P}\cdot N+\frac{N^2}{P^2}\cdot P\\ &=\Sigma\frac{N^2_i}{P_i}-\frac{N^2}{P}. \end{aligned}$$

Since the last term is a constant, Q is least if

$$Q'=\Sigma\frac{N^2_i}{P_i} \text{ is least.}$$

But
$$Q'=\sum_{i=1}^{i=48}\frac{(1+3+5+\ \ldots\ \overline{2N_i-1})}{P_i}$$

since
$$1+3+5+,\ \ldots\ +\overline{2N_i-1}=N^2_i.$$

Therefore
$$Q'=2\sum_{i=1}^{i=48}\left(\frac{.5}{P_i}+\frac{1.5}{P_i}+\frac{2.5}{P_i}+\ \ldots\ +\frac{N_i-\frac{1}{2}}{P_i}\right)$$

must be least.

If we form the table

$$\begin{array}{cccc} \frac{.5}{P_1}, & \frac{1.5}{P_1}, & \frac{2.5}{P_1}, & \frac{3.5}{P_1}, \ \ldots\\ \frac{.5}{P_2}, & \frac{1.5}{P_2}, & \frac{2.5}{P_2}, & \frac{3.5}{P_2}, \ \ldots\\ & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \frac{.5}{P_{48}}, & \frac{1.5}{P_{48}}, & \frac{2.5}{P_{48}}, & \frac{3.5}{P_{48}}, \ \ldots \end{array}$$

and choose the N smallest entries in this table and use as N_i the number of entries in the ith line, it is obvious that the quantity Q' will be as small as possible.

If we next form the table

$$\begin{array}{cccc} \frac{P_1}{.5}, & \frac{P_1}{1.5}, & \frac{P_1}{2.5}, & \frac{P_1}{3.5}, \quad \cdots \\ \frac{P_2}{.5}, & \frac{P_2}{1.5}, & \frac{P_2}{2.5}, & \frac{P_3}{3.5}, \quad \cdots \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \frac{P_{48}}{.5}, & \frac{P_{48}}{1.5}, & \frac{P_{48}}{2.5}, & \frac{P_{48}}{3.5}, \quad \cdots \end{array}$$

by using the reciprocals of the entries in the previous table, and choose the N largest entries in this new table, using as N_i the number of entries chosen in the ith line, we have exactly the same distribution as before. But this second form of this priority table shows the method identical with that of major fractions proposed by Professor Willcox. The method of minimizing the sum of the squares of the errors in the interest of a resident in a representative therefore gives another logical basis for this method. For the remainder of this paper this method, as a method of apportionment, irrespective of its logical basis, will be referred to as method W. A special advantage of a criterion for the amount of injustice involved in an apportionment lies in a possible comparison of the amount of injustice involved in apportionment for different total sizes of the House. For this purpose we define

$$Q_1 = N\sqrt{\Sigma P_i\left(\frac{N}{P} - \frac{N_i}{P_i}\right)^2}$$

as the total injustice, as a function of N. If we apply this definition for the populations as they are, we find that a total size of about* 385 gives less injustice than either larger or smaller values of N for a wide range of values.†

The application of the method of Least Squares to the expression $\Sigma\left(\frac{N_i}{N} - \frac{P_i}{P}\right)^2$ will lead to the Vinton method, as was shown by the present writer in March, 1921.‡ The proof is similar to that above and will be omitted.

* The exact arithmetic involved in this has not been carried out in complete detail.

† The idea of comparing the total amount of injustice of one apportionment as compared to that for another was first suggested by the present writer in January, 1921.

‡ Presented to the American Mathematical Society, April, 1921.

The application of Least Squares to the less significant ratios, $\frac{P}{N}$ and $\frac{P_i}{N_i}$, in the form

$$\Sigma N_i\left(\frac{P}{N}-\frac{P_i}{N_i}\right)^2$$

will minimize what might be termed the injustice to the representatives due to their having too many or too few constituents. That this method will lead to the method H was proved independently by Professor Huntington and the writer. Still other least square methods have been studied recently by Professor Huntington,* in some of which he introduces his form of relative error into the least square method, and with this form the types of expression which lead to methods H and W are interchanged.

Hence, methods H and W might perhaps be said to be the best examples of their respective types. Each is free from the Alabama paradox, and each takes care of the constitutional provision that each state must have at least one representative by assigning one representative to each state first,† arbitrarily. Each satisfies the conditions of the Constitution best when judged by its own criteria.

We must, then, in making a choice between these methods use other criteria than those contained in the methods themselves. For this purpose it is useful to examine further the implications of the phrase "among the several states according to their respective numbers." If, as is generally conceded, this means "proportional to," it carries with it the equalities,

$$(1)\ \frac{N}{P}=\frac{N_i}{N_i};\quad (2)\ \frac{P}{N}=\frac{P_i}{N_i};\quad (3)\ \frac{P}{P_i}=\frac{N}{N_i};\quad (4)\ \frac{P_i}{P}=\frac{N_i}{N};$$

$$(5)\ \frac{N}{P}=\frac{\Sigma N_i}{\Sigma P_i};\quad (6)\ \frac{P}{N}=\frac{\Sigma P_i}{\Sigma N_i}\quad (7)\ \frac{P}{\Sigma P_i}=\frac{N}{\Sigma N_i};\quad (8)\ \frac{\Sigma P_i}{P}=\frac{\Sigma N_i}{N}.$$

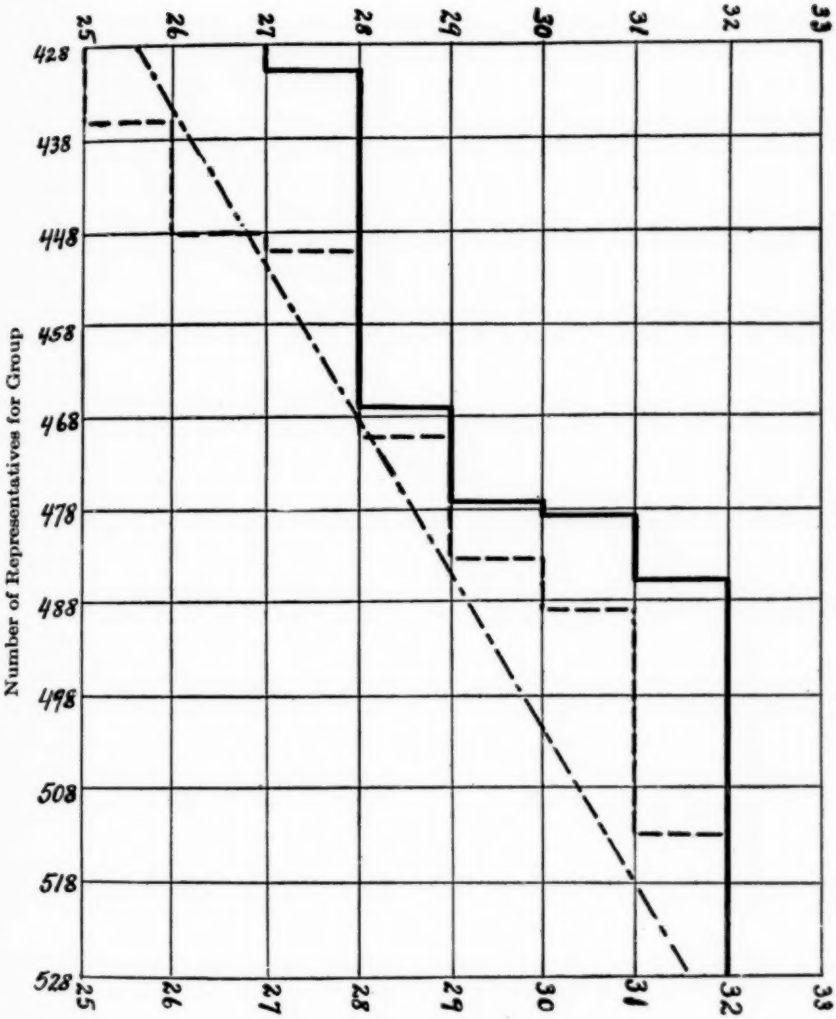
All of the equations (1), (2), (3), (4), (5), (6), (7), (8) are consequences of the idea of proportionality, and will hold strictly in an exact apportionment. When integers are used they become, in general, inequalities, and for a good apportionment all the variations from equality should be small. Their deviations from equality, however, are not all of the same magnitude. Method W is derived from (1) as the most important of (1), (2), (3), (4), and in the ordinary sense makes (1) more nearly true than does method H. It also considers (5) especially

* Presented to the American Mathematical Society, September, 1921.

† If this requirement were removed the priority table for W would still serve for an analogous method, while the use of the priority table for H would lead to slightly different results according as the error is taken relative to the smaller number, the larger number, or some average of the two.

important, and satisfies (5), (6), (7), (8) better than method H does, *no matter which criteria* are used. Although, in terms of its own criteria, method H satisfies (1), (2), (3), (4) better than method W does, it does not satisfy (5), (6), (7), (8) even closely. That this is the case can best be exhibited by specific illustration.

We will study the effect of apportionments H and W on the representation of a group of small states, consisting of Arizona, Vermont, New Mexico, Idaho, New Hampshire, Utah, Montana, Rhode Island, South Dakota, North Dakota, Maine, Oregon. These are all of the states having a population of as much as 250,000, and less than 800,000. The three still smaller states, Nevada, Wyoming, and Delaware, and their three representatives are omitted from the group on account of the fact that their populations are less than the size of the average congressional district, and hence their representatives are provided by special consitutional provision. The total population of this group of states is 6,310,819. This population multiplied by the total size of House (excluding 3) and divided by the total population (excluding Nevada, Wyoming, and Delaware) will give the true number of representatives to which this group of states is entitled. Under any *fair* method of apportionment the number of representatives assigned to the group would be sometimes more and sometimes less than the true number to which they are entitled when different totals are used for the size of the house, but the discrepancies would tend to average. In making the accompanying diagram, each total size of House from 428 to 527 inclusive was used, thus giving 100 apportionments including all the sizes which have been seriously considered by Congress. For these 100 apportionments, the true (fractional) number of representatives due the group of states is shown by the slanting long and short dash line. The heavy line shows the number assigned to the group by method H. The dash line shows the number assigned to the group by method W. The results may be summarized by saying that by method W the group is under-represented in 22 apportionments and over-represented in the remaining 78 apportionments. The average *amount* of over-representation being 0.47 of one representative or about 1.6 per cent of that due to the group. By method H the group is over-represented in every single one of the 100 apportionments, and the average amount of over-representation is more than 1.4 representatives, or about 5 per cent of the true number due to the group. Hence, if method H were to be used, the group of small states would be *certain* to be over-represented, no matter what the total number of the House, from 428 to 527. This over-representation is necessarily at the expense of under-representing the larger states. This constant



tendency to over-represent the small states (as a group) is inherent in method H and not an accident. The slight over-representation of the group by method W is an accident due to the small range of sizes chosen.

The only manner in which an apportionment can be made that will preserve the equal weight of each individual is by changing to a system whereby each representative would vote according to the number of his constituents. For example, a representative of a district containing

225,000 inhabitants would cast 225,000 votes on every roll call, the representative of a district containing 270,000 inhabitants would cast 270,000 votes, etc. This would seem at first sight to be a cumbrous method, but by installing electrical push buttons and automatic recording and adding devices (an innovation that should be installed in any case to save the waste of time in roll calls), the whole vote would be known instantly on any ballot. This, the only possible way of getting an *exactly* equal representation for each resident, could be carried out only by a change in the language, though not in the spirit, of the Constitution.

CONCLUSION

The question involved in the apportionment of representation is primarily one of constitutional law. The rôle of mathematics in the problem is to make clear the consequences of any given interpretation of the Constitution.

That there is an analogy between the distribution of money and the apportionment of representatives is indicated by their being grouped in the same clause of the Constitution, and on the basis of this analogy method W is to be preferred to method H.

If the intent of the Constitution is that any given population is to have the same expectancy of representation in the House as any other equal population, method H must be abandoned at once. Method W, however, fulfils this condition of equal expectancy much more nearly than does method H, and on this basis also is to be preferred.

A MEASURE OF DISPERSION FOR ORDERED SERIES*

By W. L. Crum, *Williamstown, Massachusetts*

It is the object of this paper to call attention to the inadequacy of the standard deviation for the study of the dispersion of a statistical series the terms of which are ordered relative to a given variable, to examine certain considerations bearing upon the dispersion in such series, and to set up tentatively a new measure particularly applicable to ordered series.

I. EXISTING MEASURES

A statistical series may be assigned to one of two broad classes according as it consists merely of a list of numbers of indefinite arrangement, or has its items ordered relative to a particular variable. Typical of the first class are the series composed of experimental measurements, and chief illustrations of the second class are to be found in historical series in the field of economic statistics. The common statistical coefficients have been developed in the study of, and are peculiarly applicable to, series of the first class; and it is a question of considerable moment whether such coefficients are equally useful in the analysis of an ordered series of the second class.

The chief measure of dispersion is the standard deviation, the square root of the mean squared deviation from the arithmetic mean. It is apparent from the method of calculation of the standard deviation that it can take no account of the arrangement of the variates.

Example (i). Consider the two series

(a) 3 7 4 6 2 5 8 7 3 4 7 3 5 8 2 6

(b) 2 3 2 3 4 5 5 6 7 7 8 7 8 6 3 4

obtained by arranging in two different orders the items

(c) 2 2 3 3 3 4 4 5 5 6 6 7 7 7 8 8.

The standard deviation for both (a) and (b) is the same as for (c), namely, 1.94. Nevertheless, inspection of the series (a) and (b), or a glance at their representation in Figure 1, is sufficient to reveal the striking difference between the two: (a) fluctuates violently, whereas (b) advances with fairly stable sweep. This coefficient fails, therefore, to distinguish the erratic series (a) from the fairly smooth (b).

* Read before the American Mathematical Society on September 9, 1921, under the title "A tentative substitute for the standard deviation in the examination of the dispersion of an ordered statistical series."

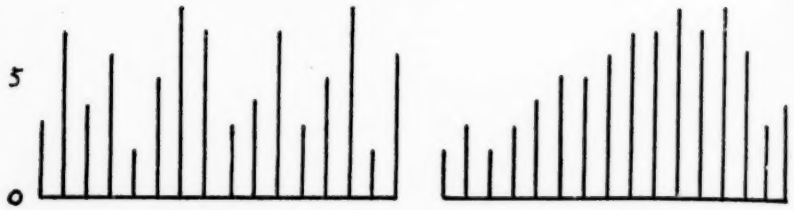


Fig I

Similar objections may be raised to the other common measures of dispersion. The coefficient of variation is derived from the standard deviation and has the same shortcoming. The computation of the mean deviation is independent of the order of arrangement of the variates, and hence it can give no better result than the standard deviation. The quartile deviation involves a grouping of the variates according to their size, regardless of the actual arrangement of the variates in their original form.

The conclusion seems warranted that existing measures of dispersion are not fitted to distinguish between such series as those of example (i). These coefficients attach the same importance to a given difference between two variates which are widely separated and two which are adjacent. They indicate the extent of the fluctuation, but fail to account for its rate. For certain purposes in the study of ordered series, particularly historical economic series, it would seem to be this very rate which it is of importance to examine and measure. Since the term *dispersion* is so widely understood in its present sense and is so generally accepted as the quality measured by the standard deviation, it may be well to avoid speaking of this rate of fluctuation as dispersion. We adopt provisionally the term *fluctuation-rate.*

II. CORRELATION COEFFICIENT

We seek now to measure the fluctuation-rate of an order series. It is suggested elsewhere* that for an ordered series possessing rectilinear trend, a more reliable measure of fluctuation than the ordinary standard deviation, σ_x, is the generalized standard deviation

$$\sigma_{x.t} = \sigma_x \sqrt{1 - r^2_{xt}}$$

since this latter in effect eliminates that part of the fluctuation due to the trend. Now the quantity r_{xt} in the above formula is the coefficient of correlation between the x_i—the series being examined—and

* Crum, W. L., *The Significance of the Partial Correlation Coefficient in the Comparison of Ordered Statistical Series Possessing Rectilinear Trends.* (Quarterly Publications of the American Statistical Association, Vol. XVII, pp. 949–952.)

the t_i—the variable relative to which the x_i are ordered. We examine in detail below the sufficiency of r_{xt} as a measure of fluctuation-rate, and any objections found will hold also against $\sigma_{x,t}$.

It would seem at first glance that the correlation coefficient r_{xt} might serve as a measure of fluctuation-rate: the smaller its value, the nearer a straight line is the join of the tops of the ordinates representing the variates, and the less is the fluctuation. It is at once necessary, however, to inquire whether a series of variates does not admit of several arrangements, differing markedly in rate of fluctuation, but all having the same value for the coefficient of correlation of the x_i relative to the t_i.

In fact, if there are N variates,

$$r_{xt} = \frac{\Sigma(t_i - T)(x_i - X)}{N\sigma_x\sigma_t}$$

where T and X are the arithmetic means and σ_t and σ_x are the standard deviations of the t_i and x_i respectively. It is evident that for a given group of x_i, N, σ_t, and σ_x will all be fixed constants, regardless of the order of arrangement of the x_i relative to the t_i. The value of r_{xt} for the various arrangements will therefore depend upon the product sum

$$Np = \Sigma(t_i - T)(x_i - X).$$

Moreover, it is easily shown that this value of Np differs from

$$A = \Sigma t_i x_i$$

only by the constant, NXT. Therefore, it is sufficient to examine the various values of A which correspond to the different arrangements in order to test the adequacy of r_{xt} as a measure of fluctuation-rate.

The question now before us is: Are there several arrangements of a given group of x_i relative to the t_i, such that for all these arrangements A is identical? If it could be shown that there is only one arrangement corresponding to a given value of A, r_{xt} would indeed be an ideal index of fluctuation-rate. Or, if it develops that all the possible arrangements giving a particular value of A are such that they do not differ from one another in the character sought to be measured—the fluctuation-rate—r_{xt} would be entirely adequate to the purpose. Unfortunately, it proves that r_{xt} falls short of meeting even this second requirement.

Without attempting at present an analytical study of the above question, we can by experiment with numerical examples bring out the essential fact about the failure of r_{xt} to discriminate properly between the various arrangements.

Example (ii). Suppose the x_i consist of the intergers 1, 2, 3, to be arranged arbitrarily relative to $t_i = 1, 2, 3$. The possible arrangements are:

		(a)	(b)	(c)	(d)	(e)	(f)
t_i	A	14	13	11	13	11	10
1	x_1	1	1	2	2	3	3
2	x_2	2	3	3	1	1	2
3	x_3	3	2	1	3	2	1

the value of A being given at the head of each series.

Series (b) and (d) each have A 13, and they are clearly similar in their fluctuation-rates. This is evident from the series themselves, or from Figure 2. Indeed, the nature of the fluctuation may be inferred from the two dotted triangles (this aspect of the question will be viewed more fully below), and the one triangle is obviously obtainable from the other by a succession of reflections and inversions. Similar remarks apply to the two series (c) and (e), for which A is 11.

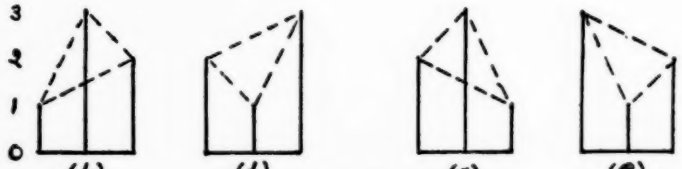


FIG. II

On the other hand, although A is 13 for (b) and 11 for (c), these two series have identical fluctuation-rates; for (c) is merely (b) in reverse order. The same is true of series (d) and (e), and the objection is even clearer in the case of (a) and (f): these pairs each have a single fluctuation-rate, but the value of A differs for the two members of the pair. It is apparent, then, that A does not have the same value for all arrangements having the same fluctuation-rate; but this is not the real question, which is whether all arrangements having the same A have equal fluctuation-rates.

Example (iii). Let the x_i be the integers from 1 to 4. The 24 possible arrangements can be classified in the following frequency table according to the value of A in the several cases:

A	20	21	22	23	24	25	26	27	28	29	30
freq.	1	3	1	4	2	2	2	4	1	3	1.

It is found, on inspection, that the series having A 20 and 30 are reverses of each other, and similarly those having A 21 and 29 are re-

verses in pairs, and so on throughout. Hence, the objection found under example (ii) arises here also, and in even more striking degree.

We examine next in detail the four series having A 27. They are:

(a) 1, 3, 4, 2; (b) 1, 4, 2, 3; (c) 2, 3, 1, 4; (d) 3, 1, 2, 4

and their diagrams appear in Figure 3. The series themselves and the diagrams show that (b) and (c) constitute a pair having equal fluctuation-rates, and so do (a) and (d); but the two pairs differ distinctly from each other. Of course, since they are the reverses of the above four series, the four series having A 23 are also grouped in pairs. Moreover, if we test the three series having A 29, it will be found that only two of them have identical fluctuation-rates.

FIG III

It appears, therefore, that even for series of the simple sort given in this last example, the value of A—and hence of r_{zt}—does not furnish an adequate measure of the fluctuation-rate: it varies when the fluctuation-rate remains unchanged, and changes when the fluctuation-rate is constant. As the series become more complicated we shall have diminishing confidence in the sufficiency of the correlation coefficient for this measurement. We must hesitate even to say that it is surely better than the standard deviation; for whereas the latter gives one value for all 24 cases in problem (iii), the correlation coefficient does indeed make a distinction; but we have seen that it makes a distinction where none exists in the fluctuation-rate.

III. THE MEAN SQUARED SECOND ORDER DIFFERENCE

We recall that one way of recognizing the equality of the fluctuation-rates of two series was by the geometric correspondence of the triangles joining the tops of the ordinates. This was most apparent in the simplest case, shown in example (ii) and Figure 2. The area of the dotted triangle suggests itself as a measure of the fluctuation-rate. In a series of more than three variates there would be a series of these triangles, each belonging to three adjacent variates: for N variates

there would be $N-2$ triangles. At any point of the ordered series the fluctuation-rate might be measured by the area of the triangle belonging to the three nearest variates; and, for the whole series, the fluctuation-rate would be indicated by an average of the $N-2$ triangular areas.

If we assume for simplicity that the t_i are integers, the area of the triangle belonging to three successive variates x_{i-1}, x_i, x_{i+1}, is

$$\text{Area} = \tfrac{1}{2}(x_{i-1}+x_{i+1}) - x_i.$$

For series (b) and (d) of example (ii) we get for the triangular area $-3/2$ and $3/2$ respectively; and these are equal if the sign is neglected. This is in accord with our inferences from the series and the figures.

In averaging the $N-2$ triangles in a series of N x_i, we may neglect the signs and take the simple arithmetic mean, or we may take the square root of the mean squared area. We choose the latter, and have for the average area:

$$\sqrt{\frac{1}{N-2}\sum_{2}^{N-1}\left(\frac{x_{i-1}+x_{i+1}}{2}-x_i\right)^2}.$$

The type expression in this summation is the square of

$$\tfrac{1}{2}(x_{i-1}+x_{i+1}) - x_i$$

which is

$$\tfrac{1}{2}(x_{i+1}-2x_i+x_{i-1}).$$

This is precisely $\frac{1}{2}$ the second order finite difference, Δ''_i. Hence the average triangular area is one-half of

$$F = \sqrt{\frac{1}{N-2}\sum_{2}^{N-1}\Delta''^2_i}$$

and, if we discard the $\frac{1}{2}$, we may take F as the new coefficient to measure the fluctuation-rate, viz., the square root of the mean squared second order difference.

We arrive at F as a measure of the fluctuation-rate by study of the deviation triangles, but the present definition in terms of the second differences gives added reason for accepting it. In fact, the first order finite difference Δ'_i is concerned with the slope of the join of two successive ordinates, whereas the second order difference Δ''_i has to do with the curvature of the join of three successive ordinates. It is this curvature, averaged over the entire extent of the series, that it is sought to measure.

It is clear that the actual calculation of F is very simple, and indeed scarcely more complicated than that of σ. Although it is necessary to calculate both the first and second differences, it is not necessary to make any correction for the position of the mean. As an example, we calculate F for the two series of example (i).

Case (a)

x_i	3	7	4	6	2	5	8	7	3	4	7	3	5	8	2	6
Δ'_i	4	−3	2	−4	3	3	−1	−4	1	3	−4	2	3	−6	4	
Δ''_i	−7	5	−6	7	0	−4	−3	5	2	−7	6	1	−9	10		

$F = 5.85$

Case (b)

x_i	2	3	2	3	4	5	5	6	7	7	8	7	8	6	3	4
Δ'_i	1	−1	1	1	1	0	1	1	0	1	−1	1	−2	−3	1	
Δ''_i	−2	2	0	0	−1	1	0	−1	1	−2	2	−3	−1	4		

$F = 1.81$

Evidently F distinguishes readily the erratic series (a) from the fairly regular (b).

We turn now to those series of example (iii) for which A is 27. The values of F for the four series are:

(a) 2.24; (b) 4.12; (c) 4.12; (d) 2.24.

We recall from Figure 3 that (c) is the "reflection" of (b), and has the same fluctuation-rate. This fact, which was not, to be sure, out of accord with the single value 27 for A, is borne out by this new coefficient. The same is true of (a) and (d). On the other hand, the distinction between the two pairs, which was not indicated by any change in A, is clearly shown by the differing value of F. Moreover, a calculation of the values of F for the four series having A 23 shows the same values as above, and the series in the A 23 group which are the reverses of (a) and (d) in the A 27 group have also F equal to 2.24, and similarly for the other pair. Similar tests apply throughout, and we may, therefore, say that both the objections raised against A are met by F.

Although we are not yet in a position to say with finality that F furnishes the best measure of the fluctuation-rate in an ordered statistical series, the tests which we have made above seem to indicate that it may be accepted tentatively as such measure. It is hoped that further study of the coefficient F will serve to develop more fully the real significance of the square root of the mean squared second order difference, and perhaps result in devising another measure even more efficacious than F.

PHYSICAL DEVELOPMENT OF VASSAR COLLEGE STUDENTS, 1884–1920

By Mabel Newcomer, *Vassar College*

It is a matter of common belief that women are growing larger. In order to ascertain whether such a change—or any change—in the physical development of women is taking place, the class in statistics of the economics department of Vassar College has made a study of the physical development of college women over a period of thirty-seven years.

The data for the investigation were obtained from records of the department of physical training, covering the period from 1884 to 1920 inclusive. Those measurements selected, upon consultation with the director of physical training, as indicative of the physical development of students, and also as comparable through the entire period, were height, weight, girth of waist, and capacity of lungs. The age and the number of sports in which the student claimed some proficiency were also tabulated as possibly offering a partial explanation of any change in development which might be found. Only the records of freshmen entering the regular academic course were used, and from these only the measurements made during the first physical examination, given shortly after the student enters college in the fall, were taken. In some instances records have been included which lack one or more of the measurements. For instance, none of the 1918 records includes the girth of waist, and in 1897 only three measurements were recorded for capacity of lungs. Consequently the frequencies vary somewhat for the different measurements. The total number of records used was 7,077, and in all of these cases the age and at least two other measurements were given.

During the earlier years, 1884–90, the examinations for each year were made by a single instructor, but not by the same instructor each year, so that there may have been slight variations in the methods of measuring employed from year to year. During the thirty years, 1891–1920, the examinations were made usually by more than one instructor in each year; but for the entire period they have been under the supervision of the present director of physical education, and there is no question as to the uniformity, and consequent comparability, of the measurements for this period.

Both the height and the girth of waist were recorded to the nearest

tenth of a centimeter, and the arithmetic means were computed from tables in which the class interval was one centimeter. Weights were recorded to the nearest quarter of a pound. The measurements were grouped in classes of five pounds each for computing the mean. The capacity of the lungs was recorded to the nearest cubic centimeter, and the average was computed from a table grouping the measurements in classes of ten cubic centimeters. The averages referred to in the text are in each case the averages of five year periods because of their greater st ability.

The increase in height for the period covered is considerably more than an inch. During the last thirty years the arithmetic mean has increased from 160.4 cm. to 163.7 cm., an increase of 3.3 cm. or 1.3 inches; the median has increased from 160.4 cm. to 163.6 cm. The yearly averages show an increase in excess of an inch and a half.

TABLE I
ARITHMETIC MEANS OF HEIGHTS OF VASSAR COLLEGE STUDENTS, 1884–1920

Year	No. of cases	Mean (cm.)	Standard deviation	Year	No. of cases	Mean (cm.)	Standard deviation
1884.....	30	161.4	5.8	1903.....	207	163.0	5.7
1885.....	39	159.7	5.0	1904.....	247	162.7	5.7
1886.....	31	158.9	4.4	1905.....	208	162.1	5.5
1887.....	44	161.5	4.7	1906.....	160	162.6	5.7
1888.....	52	159.9	4.1	1907.....	295	162.1	5.7
1889.....	59	160.7	6.0	1908.....	286	163.1	5.6
1890.....	51	161.0	5.1	1909.....	260	163.2	5.5
1891.....	113	160.4	5.3	1910.....	267	162.0	5.4
1892.....	127	160.1	5.8	1911.....	256	163.3	5.7
1893.....	129	160.3	5.5	1912.....	276	162.8	6.2
1894.....	105	160.9	5.3	1913.....	296	163.0	5.5
1895.....	146	160.2	5.9	1914.....	343	162.9	5.8
1896.....	164	161.3	5.3	1915.....	303	163.1	5.3
1897.....	201	161.2	5.7	1916.....	227	162.9	5.2
1898.....	185	161.3	5.6	1917.....	241	163.6	5.6
1899.....	218	161.6	5.8	1918.....	228	163.9	5.3
1900.....	222	162.0	5.4	1919.....	242	163.7	5.9
1901.....	290	162.3	5.8	1920.....	296	164.4	6.1
1902.....	220	161.8	5.7	Total....	7,064	162.4	5.7

TABLE II
ARITHMETIC MEANS OF HEIGHTS OF VASSAR COLLEGE STUDENTS AVERAGED BY FIVE-YEAR GROUPS, 1884–1920

Years	No. of cases	Mean (cm.)
1884–1890	306	160.5
1891–1895	620	160.4
1896–1900	990	161.5
1901–1905	1,172	162.4
1906–1910	1,268	162.6
1911–1915	1,474	163.0
1916–1920	1,234	163.7

The increase in weight is also marked, being six and one-half pounds in the last thirty years. The arithmetic mean has increased from

118.2 pounds in the years 1891–95 to 124.7 in 1916–20. The medians for the same periods are 117.1 and 123.4 respectively. These measurements include the weight of the examining robe, which is at present one pound and two ounces. These robes weighed slightly more in the earlier years.

TABLE III

ARITHMETIC MEANS OF WEIGHTS OF VASSAR COLLEGE STUDENTS, 1884–1920

Year	No. of cases	Mean (pounds)	Standard deviation	Year	No. of cases	Mean (pounds)	Standard deviation
1884.....	30	123.8	8.7	1903.....	207	124.6	7.5
1885.....	39	115.7	6.4	1904.....	246	123.1	7.4
1886.....	31	114.1	5.3	1905.....	209	121.4	7.8
1887.....	44	118.5	5.6	1906.....	161	124.3	7.3
1888.....	53	118.4	6.6	1907.....	295	123.6	7.5
1889.....	58	119.1	6.2	1908.....	282	122.4	7.9
1890.....	51	117.0	5.5	1909.....	265	124.4	8.0
1891.....	113	118.0	6.5	1910.....	266	122.8	7.5
1892.....	127	116.6	5.4	1911.....	257	126.4	9.3
1893.....	127	119.2	6.1	1912.....	277	125.2	7.8
1894.....	106	117.3	6.4	1913.....	299	124.8	7.3
1895.....	147	119.5	6.9	1914.....	341	122.1	6.8
1896.....	164	118.9	6.5	1915.....	307	124.3	7.1
1897.....	197	118.9	6.3	1916.....	227	124.5	7.0
1898.....	186	118.7	7.6	1917.....	239	122.3	6.4
1899.....	217	121.9	7.7	1918.....	229	124.4	6.9
1900.....	223	121.9	6.8	1919.....	242	126.1	7.0
1901.....	282	120.9	7.2	1920.....	296	125.8	6.9
1902.....	220	121.3	7.4	Total....	7,060	122.5	7.4

TABLE IV

ARITHMETIC MEANS OF WEIGHTS OF VASSAR COLLEGE STUDENTS AVERAGED BY FIVE-YEAR GROUPS, 1884–1920

Years	No. of cases	Mean (pounds)
1884–1890	306	118.1
1891–1895	620	118.2
1896–1900	987	120.3
1901–1905	1,164	122.2
1906–1910	1,269	123.5
1911–1915	1,481	124.4
1916–1920	1,233	124.7

The girth of waist does not show this regular increase. In fact there is a fairly steady decrease in this measurement from 1884 to 1904, and a regular increase from 1904 to 1920. The change in the type of clothing worn, together with the increase in weight, readily account for the recent increase; but neither of these explains the decrease of the first twenty years. The only explanation which suggests itself is that the earlier type of college woman was less subservient to fashions than those during the later years when higher education of women had become more widespread. The increase in 1916–20 over 1884–90 is just nine-tenths of an inch; and the increase over 1901–05 is nearly

two inches. The arithmetic mean is 62.9 cm. for 1884–90. It drops to 60.2 in 1901–05 and rises again to 65.2 in 1916–20. The median for the same years is 62.6, 59.5, and 64.8 respectively.

TABLE V
ARITHMETIC MEANS OF GIRTH OF WAIST OF VASSAR COLLEGE STUDENTS, 1884–1920

Year	No. of cases	Mean (cm.)	Standard deviation	Year	No. of cases	Mean (cm.)	Standard deviation
1884	30	65.2	4.6	1903	206	60.5	4.9
1885	39	63.2	3.6	1904	248	59.0	4.8
1886	31	62.7	6.4	1905	208	60.0	4.6
1887	44	62.7	4.3	1906	160	61.5	5.4
1888	53	62.9	3.8	1907	293	61.2	5.5
1889	59	62.7	3.9	1908	283	61.0	5.1
1890	51	62.1	4.9	1909	267	63.8	5.5
1891	112	63.0	4.5	1910	263	63.3	5.0
1892	127	62.6	3.6	1911	254	63.8	4.9
1893	127	62.7	4.0	1912	274	63.9	5.2
1894	107	62.5	5.1	1913	297	63.9	4.5
1895	146	62.7	4.4	1914	340	65.4	4.4
1896	160	59.6	4.2	1915	307	65.6	5.0
1897	199	60.2	4.1	1916	227	65.6	4.7
1898	183	60.9	4.8	1917	239	64.9	4.3
1899	216	61.2	4.8	1918	....	....	...
1900	223	61.0	4.4	1919	240	65.1	4.4
1901	283	60.6	4.5	1920	296	65.3	4.1
1902	220	60.8	4.5	Total	6,812	62.7	5.1

TABLE VI
ARITHMETIC MEANS OF GIRTH OF WAIST OF VASSAR COLLEGE STUDENTS AVERAGED BY FIVE-YEAR GROUPS, 1884–1920

Years	No. of cases	Mean (cm.)
1884–1890	307	62.9
1891 1895	619	62.7
1896–1900	981	60.6
1901–1905	1,165	60.2
1906–1910	1,266	62.2
1911–1915	1,472	64.5
1916–1920	1,002	65.2

The measurements of lung capacity are of doubtful value. The measurements of the spirometer often vary considerably for the same individual. Also, there have been changes in the type of spirometer used, and there is no record of when these changes were made. The wide fluctuations in the measurements during the earlier years, particularly from 1894 to 1898, can be attributed only to faults of measurement, probably through some defect of the spirometer. With the last twenty years, however, there is more regularity; and the same increase is apparent here that is found in the other measurements. The arithmetic mean increases from 166 cc. in 1901–05 to 180 cc. in 1916–20. The median increases in this same period from 164 cc. to 179 cc.

TABLE VII

ARITHMETIC MEANS OF CAPACITY OF LUNGS OF VASSAR COLLEGE STUDENTS, 1884–1920

Year	No. of cases	Mean (cc.)	Standard deviation	Year	No. of cases	Mean (cc.)	Standard deviation
1884	30	177	32	1903	208	170	23
1885	39	168	23	1904	247	169	23
1886	31	154	24	1905	206	169	27
1887	44	156	32	1906	161	166	26
1888	53	148	28	1907	295	167	24
1889	59	156	28	1908	283	169	25
1890	51	161	29	1909	264	169	24
1891	108	149	34	1910	265	167	26
1892	127	164	28	1911	256	169	27
1893	126	159	32	1912	263	168	26
1894	107	96	36	1913	300	180	25
1895	147	119	32	1914	339	172	27
1896	160	117	28	1915	304	174	25
1897	198	100	21	1916	226	179	24
1898	3	62	10	1917	239	179	27
1899	55	158	27	1918	225	176	26
1900	223	159	22	1919	241	182	27
1901	282	161	24	1920	293	182	26
1902	218	161	23	Total	6,676	164	32

TABLE VIII

ARITHMETIC MEANS OF LUNG CAPACITY OF VASSAR COLLEGE STUDENTS, AVERAGED BY FIVE-YEAR GROUPS, 1884–1920

Years	No. of cases	Mean (cc.)
1884–1890	307	159
1891–1895	615	138
1896–1900	639	130
1901–1905	1,161	166
1906–1910	1,268	168
1911–1915	1,462	173
1916–1920	1,224	180

The very considerable increase in all of these measurements, as well as the uniformity of growth, leaves no reasonable doubt that there has been a decided increase in the physical development of Vassar College students. Accepting this, two questions at once present themselves. First, are these changes typical of all women? Second, what is the cause of such growth? Such data as are available indicate that such changes are typical of college women, at least.*

The investigation does not show why there has been such an increase, although certain conclusions may safely be drawn from the results. The mean age of entering students has decreased from 19.0 years in

* More limited investigations were made at Stanford University and at Smith College last year. That made at Stanford University was of the heights of entering women students for a period of thirty years. (Mosher, C. D., "Concerning the Size of Women." *California State Journal of Medicine*, Feb., 1921.) The Smith College investigation covers both heights and weights, beginning with the class of 1903. (*New York Herald*, May 8, 1921.) Both of these show results for the period and measurements covered very similar to the Vassar College investigation.

1891–95 to 18.2 years in 1916–20. This fact indicates that an even greater increase in physical development would be found if the cases were grouped and studied according to age. The change in age of entrance is not great, however, and at all times is closely concentrated around eighteen years. Of the entire 7,077 cases, 6,009 fall in the three years over 17 and under 20; and 6,805 in the five years over 16 and under 21.

There is no record of race, which might have a marked influence on size, further than that offered by birthplace, which is entirely inadequate. The number of foreign born is always small, and a large part of these are of American stock.

A study of residence was made as perhaps throwing some light on the problem. It has been suggested that the number of students from western states was perhaps increasing, and that western students might be larger. Two groups were chosen: those students in college in 1890–91, which includes most of those students entering from 1887 to 1890, and those in college in 1920–21, or those entering from 1917 to 1920. The following distribution (Table IX) was found, showing a slight shift westward. These changes seem too small to be significant. Further, it might be noted that the height of the Vassar student is slightly greater throughout than that of the student at Stanford, where the proportion of western and eastern students is very nearly reversed.

TABLE IX

PERCENTAGE DISTRIBUTION OF STUDENTS ACCORDING TO RESIDENCE, 1890–91 AND 1920–21

States	1890–91	1920–21
New England	18.5	20.4
Middle Atlantic	46.4	47.0
East North Central	19.4	15.1
West North Central	4.5	4.9
South Atlantic	1.4	4.4
East South Central	4.5	1.4
West South Central	.5	.7
Mountain	.5	1.7
Pacific	4.1	2.3
Foreign	.5	2.0

The study of the number of sports in which the students engaged offers a positive contribution to the problem. Beginning in 1896 the students were asked to state in what sports they had engaged regularly before entering college, and a record h' been kept of these. Although the interpretation of this question doubtless varies greatly with individuals, there is no reason to expect more cases of overstatement of the number of sports in recent than in earlier years, and the

regularity and the size of the increase cannot be ignored or explained otherwise than as indicating a decided increase in the number of sports engaged in. And it seems reasonable to suppose that this is an important factor in the increase in size—how important is a matter to be determined by instructors in physical education and by others. In 1896–1900, 26.5 per cent of the students reported no sports at all, whereas in 1916–20 those reporting no sports constituted 0.6 per cent of the whole. The maximum number of sports reported by any student in the earlier period was eight; in the later, seventeen.

TABLE X

AVERAGE NUMBER OF SPORTS ENGAGED IN BY VASSAR COLLEGE STUDENTS, 1896–1920, BY FIVE-YEAR GROUPS

Years	No. of cases	Mean
1896–1900	992	2.0
1901–1905	1,167	3.1
1906–1910	1,272	3.9
1911–1915	1,482	6.3
1916–1920	1,236	9.2

Whether such increases in physical development will continue is an interesting matter for speculation: but it is for speculation only until the causes of increase are more definitely ascertained.

NOTES

METHODS USED IN MEASURING UNEMPLOYMENT

By Ernest S. Bradford*

Unemployment, as the term is used in this discussion, relates to the manufacturing and mechanical industries, including the building and hand trades, and, to a less extent, to transportation and mining. This is partly because the records kept by responsible statistical bodies in the United States are confined mainly to wage earners in manufactures and in building. The data available regarding unemployed persons in agriculture, retail and wholesale trade, domestic service, and the clerical occupations are in most instances too meager to constitute a substantial body of reliable statistics.

EMPLOYED PERSONS

In studying the unemployed it is necessary first to separate those gainfully employed into two classes, (1) employees, and (2) employers and those who are self-employed. Employees are those who work for an employer, the latter supplying the equipment, including factory, machinery, and tools, the former performing a certain part of the useful work required in the conversion of materials, and receiving for their services wages or salary. Idleness affects persons thus employed more seriously than employers or the self-employed. The farmer, the professional man, and other independent operators also frequently have greater resources than the wage earner. The seriousness of a condition of wide unemployment, therefore, is of first concern to those who are employees.

Of the total number of persons gainfully employed in 1920, amounting to between 41,000,000 and 42,000,000, according to preliminary figures based on returns from 32 states, there are possibly 30,000,000 who may properly be called employees.† Of this number between 15,000,000 and 16,000,000 are found in the group of manufacturing and mechanical industries, extraction of minerals, and transportation.

* Mr. Bradford was a member of the Economic Advisory Committee of the President's Conference on Unemployment.

† Dr. Willford I. King estimates the number of employees in 1918 as 30,224,000, excluding about 9,750,000 entrepreneurs and the members of farmers' families who are working on their home farms. The writer's estimate is about the same for 1920, based on a detailed splitting of the occupations groups of the census into employees and others.

These are the persons whose unemployment we are particularly concerned in measuring.

MEASURING UNEMPLOYMENT

There are two principal methods of measuring unemployment: (1) the method of counting or directly estimating the number of unemployed; (2) the indirect method of estimating the reduction in the number of persons employed at certain times or between one date and another. It is assumed in the latter case that the difference between the maximum and the minimum number of employees (or persons on pay-rolls) represents the unemployed, at least at the particular time in question, or during a certain period. Figures arrived at by both of these methods were taken into account in estimating the number at present unemployed for the President's Conference on Unemployment, recently held in Washington.

The same two methods may also be used in measuring the number of unemployed over a series of years, and in arriving at the amount of normal or average unemployment. The records available in this country for a study of unemployment fall into one or the other of these two classes.

DIRECT MEASUREMENT OF UNEMPLOYMENT

The direct method of attempting to secure the number of unemployed persons has been used by the Massachusetts Department of Labor and Industries since 1908, and is based on the records of unemployment among organized wage earners. The number of labor organizations reporting to the Bureau of Statistics of the State Department of Labor and Industries includes about 250,000 members, roughly 75 per cent of the total number of union wage earners in the state. This is regarded as a large enough proportion to be typical of the organized wage earners of the entire state. The figures appear to have been collected on a uniform plan, year after year, and to be compiled with care. There is no reason to believe that the percentage of unemployment if taken in the middle instead of at the end of each quarter, or even at the end of each month, would materially change the unemployment average for the year. It is also believed that the percentage of union unemployment represents fairly well the unemployment of all industrial labor in this state—at least unemployment is not less among the non-union unskilled workers than among the organized workers, who are skilled for the most part, and who have the help of the union, when out of work, in finding a new job.

There is some evidence that the percentages of union and non-union

unemployment are about the same in the leading industries of Massachusetts during the present depression. A census of the reduction in employment in textiles, boots and shoes, and metals and machinery, taken December 18, 1920, showed closely similar percentages to those of union unemployment in the same industries at the close of the quarter, December 31, 1920.

Similar figures of union unemployment in the state of New York were collected from representative labor unions each month from 1904 to the middle of 1916, when this series was discontinued and the number of employees on pay-rolls of representative factories was collected instead. The number of members of unions reporting as to idleness was gradually increased from about 100,000 in 1904 to about 200,000 in 1916, there being in 1914 about 140,000 in unions reporting. This is regarded as a large enough number to be roughly typical of industry in the state, particularly since the localities and the unions were selected with this in mind. The figures are therefore believed to be fairly representative of unemployment as a whole in New York, and also of unemployment in at least four leading industries.

In order to translate unemployment percentages into a unit which shall have significance and shall also be comparable with other percentages, it is necessary to have a definite time element present. If 10 per cent of the wage earners are unemployed at the end of each month, even though the actual persons unemployed are different from one month to the next, the average percentage of unemployment for the year is 10 per cent. It is assumed that the average of the percentages of unemployment taken at the end of each month will not differ greatly from the average of percentages taken at the middle of each month, or at any other time during the month.

The good points of the union unemployment figures of Massachusetts and New York are:

1. They have been collected regularly.
2. They have been collected by recognized statistical bureaus.
3. They have been obtained from secretaries of labor unions who are presumably well informed as to employment of members (except in the largest unions), although they are not statisticians.
4. Even if not absolutely correct, the figures represent relatively correct fluctuations from month to month or quarter to quarter.

On the other hand:

1. The figures may be exaggerated by union secretaries, giving higher unemployment estimates during depressions and lower estimates during good times than actual conditions warrant.
2. They may be intentionally exaggerated.

3. They may be too low to hold true of labor in general, since they apply only to union labor, which is skilled labor and not so often out of work as the non-union and unskilled.

These last points tend to offset one another. Further, experience in dealing with the less exact types of statistics in business and industry tends to the belief that these figures are fairly correct, especially in Massachusetts.

In using these figures it was assumed that (a) the unemployment of union industrial workers is roughly typical of general industrial unemployment in these states; and (b) these states are probably typical of other large industrial states.

Examination of the industries involved in these states does not reveal any reason to believe that unemployment would differ markedly in industries not represented.

REDUCED EMPLOYMENT AS A MEASURE OF UNEMPLOYMENT

The other principal method of estimating unemployment is to measure the differences between two or more employment levels. The state of New York, for example, since 1914 has collected figures showing the number of persons on the pay-rolls of certain representative factories, numbering approximately 1,600 plants and averaging between 500,000 and 600,000 wage earners—more than one-third of the factory workers of the state. The index number used in this method, starting with 100 per cent in July, 1914, reached 125 in March, 1920, and fell to 90 in June, 1921; 88 in July; 88.1 in August; and rose to 91.5 in September.

The state of Wisconsin has been using a similar system since 1915, and its employment curve parallels closely that of New York. About a third of the industrial wage earners of the state are represented.

The state of New Jersey has kept a record of the number of wage earners employed in its industries from 1898 up to and including 1918. These figures cover almost all of their industrial wage earners, and thus afford an excellent index of employment for that state. The figures for 1919 are not yet available because they were collected by the United States Census of Manufactures. The figures for 1920 are also still lacking.

Massachusetts takes an annual census of manufactures which shows the number of wage earners employed each month by the industries of the state. This furnishes also an employment curve of similar significance.

The Bureau of Labor statistics since October, 1915, has published in the *Monthly Labor Review* the number of persons on the pay-rolls of certain representative factories, numbering about 1,400 and repre-

senting from 600,000 to 800,000 employees. Identical establishments are compared as between any two consecutive months, but they are not identical for all the months of the year, as are the New York pay-roll figures. They cover 14 industries and more than 15 states, supplying information over a wider range of territory than any single state. The industries covered are well known to readers of this magazine. This supplements the data of the several states.

The method used by the United States Employment Service in January, 1921, and again in September, 1921, consisted in securing directly from certain states and cities the reduction in the number of persons reported as employed, as between January, 1920, and January, 1921. The value of this estimate naturally depends upon the care with which the figures were gathered in each state—whether by trained statisticians, and by what methods. It seems clear that information collected by a state or a federal bureau of statistics of industry or labor in a methodical and consistent manner, along lines previously found to yield dependable figures, as in Massachusetts or New York, should be regarded as more reliable than miscellaneous estimates obtained by a Chamber of Commerce or a Board of Trade, inexperienced in this field, through persons varying in statistical abilities and opportunities. Whether or not the estimate of 3,473,000 persons unemployed as of January, 1921, arrived at in this manner, is a close one depends upon the extent to which the data were collected through trained and responsible persons and on how far miscellaneous guesses were incorporated along with the estimates of properly organized statistical bureaus.

The monthly figures of unemployment published in the *Industrial Employment Survey Bulletin* (which are not to be confused with the foregoing estimate of reduced employment as of January, 1921, compared with January, 1920) and the curve which accompanies them were intended to indicate the trend of industrial employment from month to month. Pay-roll data were secured from about 1,400 concerns, usually employing 500 or more persons each, located in 65 industrial centers and representing in the aggregate 1,500,000 wage earners at the lowest point, July, 1921. The first figures in this series give the total number of workers on pay-rolls in such plants in February, 1921, compared with January, 1921, and they have been carried on currently each month since. They cover concerns in 14 groups of industries, and follow closely the census classification. They also show increases and decreases in factory employment by cities as well as by groups of industries. This method is similar to that employed in New York state and in Wisconsin, but covers a wider area and takes in only the larger concerns.

Several of these ways of measuring unemployment result in figures which agree rather closely with one another as to the extent of the reduction in employment between January, 1920, and the present time. The New York index of factory employment dropped from 123 in January, 1920, to 88.1 in August, 1921—35 points, or 28 per cent. The index of the Bureau of Labor Statistics dropped from 106 in January to 82 in July, 1921—24 points, or 23 per cent. Unemployment in Massachusetts rose from 4.1 at the end of December, 1919, to 25.9 on July 1, 1921, or 21.8 more than at the beginning of 1920.

All of these figures point to the same approximate conclusion that between January 1st, 1920, and September 1st, 1921, employment fell about 25 per cent. Applying this percentage to the 12,000,000 employees engaged in manufacturing and mechanical occupations, we find about 3,000,000 fewer factory wage earners employed in September, 1921, than in January, 1920. If we add mining, in which the reduction in employment in January, 1921, from January, 1920, was, say, 250,000, and railway transportation, in which the reduction was about 450,000, we have a reduction of 700,000 more. These figures do not take into account a considerable number of persons who were changing jobs, sick, or on strike; they represent reduced employment rather than actual unemployment, which latter is somewhat greater. Even a small addition to this 3,700,000 would bring the total up to about 4,000,000 fewer employed, not including persons in trade, agriculture, domestic service, or clerical occupations. The number which should be added from these fields cannot be estimated because of the lack of data. The increase which should be made on this account will be offset to some extent by the fact that we are measuring from an abnormally high peak, namely, January, 1920, when many persons, including women and boys not normally employed, were on pay-rolls. It is also offset partly by the fact that some persons taken off factory pay-rolls are taking other jobs and are therefore not unemployed; there is little evidence, however, that this movement has been carried as far during 1921 as many seem to believe. The evidence points to an aggregate of reduced employment of about 4,000,000 in the manufacturing industries, mining and transportation, without attempting to estimate the situations in other occupations, in which, however, there is unquestionably a much smaller proportion of reduced employment.

CENSUS OF MANUFACTURES

The figures of the United States Census of Manufactures, gathered every five years, showing in each industry the average number of persons employed on the fifteenth day of each month, or on the nearest

representative day, are helpful in this connection. The available data indicate the up and down movements in employment, and hence in unemployment, in general and for particular industries during the year for which they are given. For the four intervening years between censuses, however, figures are missing.

In a general way, it may be said that the method of measuring unemployment through reduced employment—that is, from the top down—fails to include the percentage of those who are always out of work. By this is meant not the unemployable, but, as already stated, those persons (of whom there is always a certain per cent) who are changing jobs and are on no pay-roll, and those who are sick or on strike. These are not always the same individuals, but the total average percentage seems to remain fairly constant.

On the other hand, the method of measuring unemployment by considering the unemployed among organized workers—from the bottom up, as it were—tends to exaggerate the existing conditions, the secretary of the union being inclined to report fewer persons unemployed during good times than there actually are, and more unemployed than there actually are during seasons of depression. However, it is not quite correct to use the term "from the bottom up" since there is no measure of the unemployment among a great mass of unskilled and unorganized workers, who are frequently referred to as being at the bottom of industry.

The method of measuring unemployment through fluctuations in the total number of employed entered on factory pay-rolls, as reported by the United States Census of Manufactures, is open to another objection, namely, that when all persons in industry are taken as a whole, the range from the maximum to the minimum number employed is much less than it would be if each industry were taken separately and the separate differences between maximum and minimum were then all averaged. In other words, if industry is regarded as a single labor reservoir, the range in 1914 and in 1909 between the high and the low level is only about 8 per cent; whereas if it is regarded, more accurately, as a series of smaller reservoirs, passage from one of which to another is not free, the average of the ranges runs around 15 per cent. If this process is pursued further and each plant or establishment is taken as the employing unit, which is still nearer to existing conditions—presenting, in effect, a much larger number of still smaller reservoirs or employing units of labor—the average of the difference in levels rises still higher to an undetermined number, equivalent to perhaps 20 or 25 per cent. In terms of *constant unemployment*, the percentage is about half as great as this difference between high and low.

In a large country of such extended industrial interests as the United States, the lack of adequate statistics on employment matters is surprising. Only by the most persistent and painstaking analysis can existing data be made to yield an explanation likely to fit the facts. At a time like the present when we need to know how much unemployment there is, how it compares with past unemployment, how much is seasonal, how much is cyclical or depressional, and how rapidly it is growing or waning, we are confronted with huge gaps in our statistical knowledge, to be bridged only by those who have detailed information regarding conditions in a number of particular industries. The first need, therefore, is for better unemployment data.

Suggestions for better statistics of unemployment may be summarized as follows:

1. In every industrial state records should be kept both of employees on pay-rolls and of union unemployment in order to supply an adequate basis of fact in both respects. These records should cover not only the manufacturing and building trades, but also mining, transportation and trade.

2. These figures should be collected regularly, preferably by *months* rather than by quarters.

3. They should be collected by some responsible statistical body, preferably a public bureau, organized to handle industrial or labor statistics.

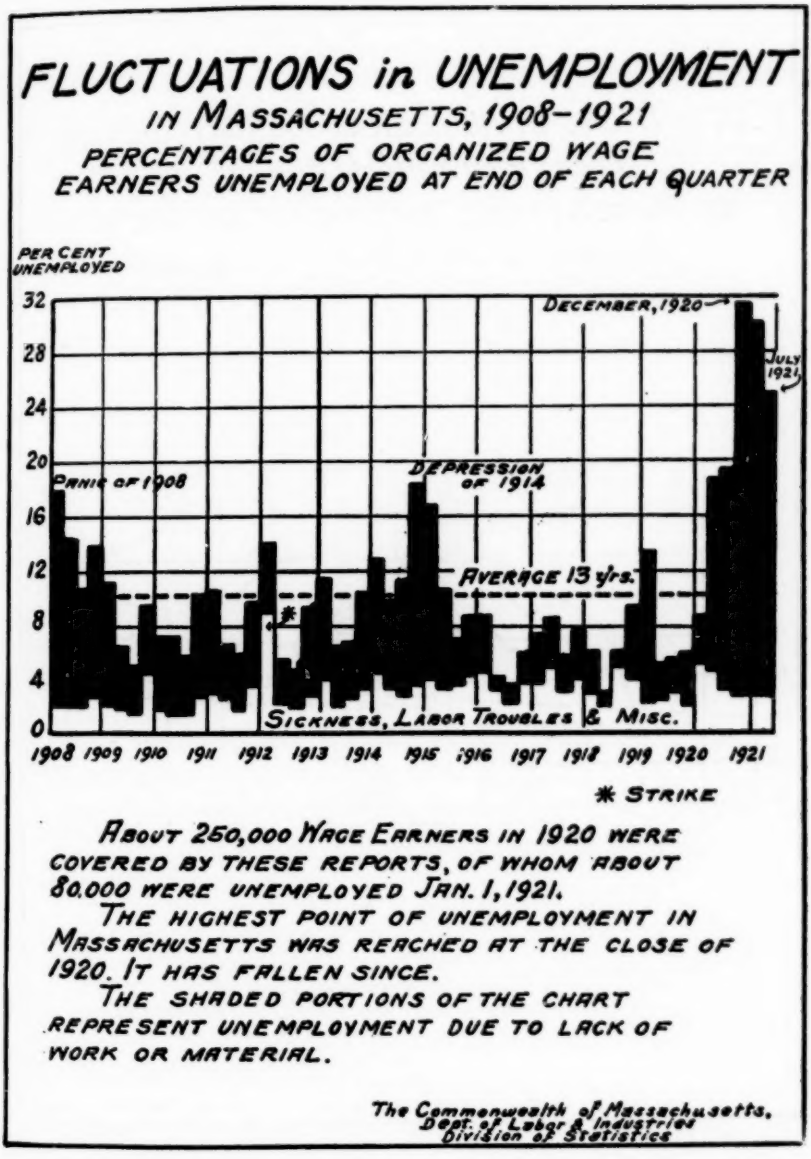
4. Unemployment due to sickness and labor disputes should be separated from that due to lack of work or material.

5. The figures should show partial unemployment (part time employment) as well as complete unemployment.

6. The data should be published frequently in order to keep the employer, the employee, and the public constantly informed as to the state of business and of unemployment. There is no better barometer of industrial conditions than an unemployment index. It also acts as a warning as to the degree of social ill-being existing when men are out of work, hungry, and discontented.

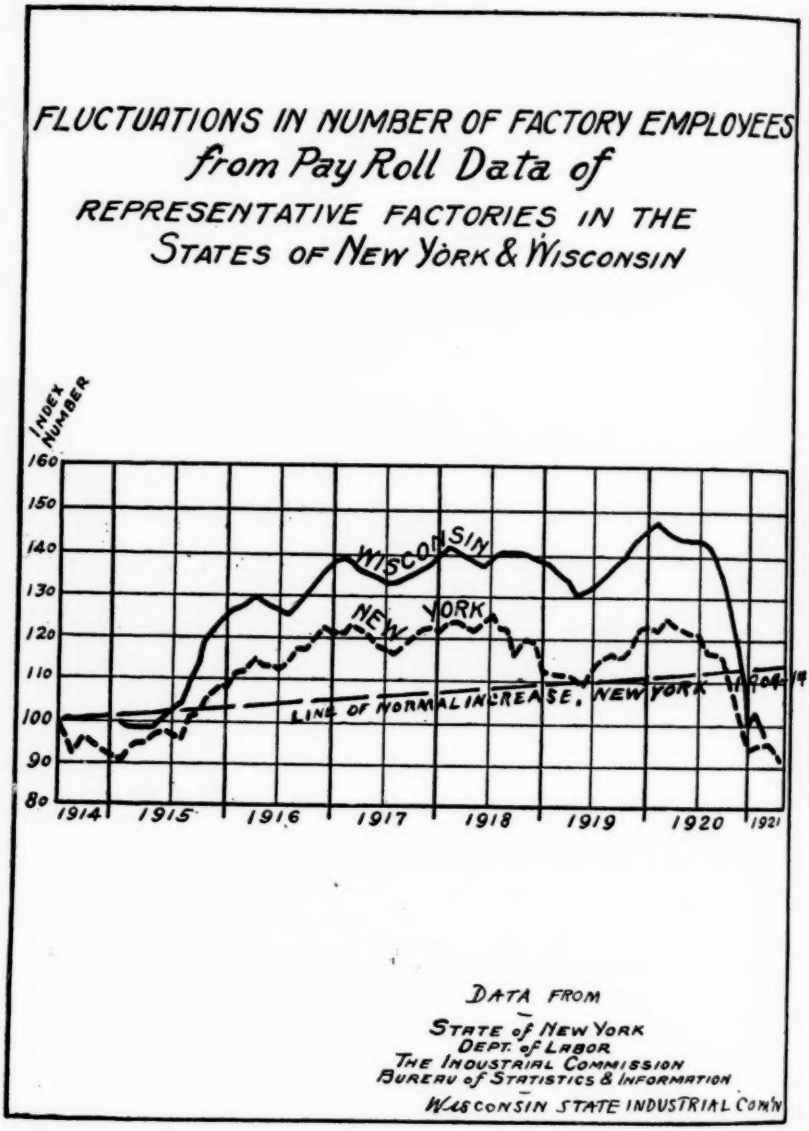
7. Figures of wages and hours of work can be collected to advantage at the same time; wages multiplied by hours of employment show the wage earner's income and therefore his purchasing power.

8. Only one report or set of figures should have to be prepared by the individual employer or union. In order to reduce the burden of compiling data, when the federal government and the state both require information, arrangement should be made for one office to collect the data and supply it to the other, as is now done, for example, by the state of Massachusetts.


FLUCTUATIONS in UNEMPLOYMENT
IN MASSACHUSETTS, 1908–1921
PERCENTAGES OF ORGANIZED WAGE EARNERS UNEMPLOYED AT END OF EACH QUARTER
PER CENT UNEMPLOYED
32
28
24
20
16
12
8
4
0
DECEMBER, 1920
JULY 1921
PANIC OF 1908
DEPRESSION OF 1914
AVERAGE 13 yrs.
SICKNESS, LABOR TROUBLES & MISC.
1908 1909 1910 1911 1912 1913 1914 1915 1916 1917 1918 1919 1920 1921
* STRIKE
ABOUT 250,000 WAGE EARNERS IN 1920 WERE COVERED BY THESE REPORTS, OF WHOM ABOUT 80,000 WERE UNEMPLOYED JAN. 1, 1921.
THE HIGHEST POINT OF UNEMPLOYMENT IN MASSACHUSETTS WAS REACHED AT THE CLOSE OF 1920. IT HAS FALLEN SINCE.
THE SHADED PORTIONS OF THE CHART REPRESENT UNEMPLOYMENT DUE TO LACK OF WORK OR MATERIAL.
The Commonwealth of Massachusetts, Dept. of Labor & Industries Division of Statistics

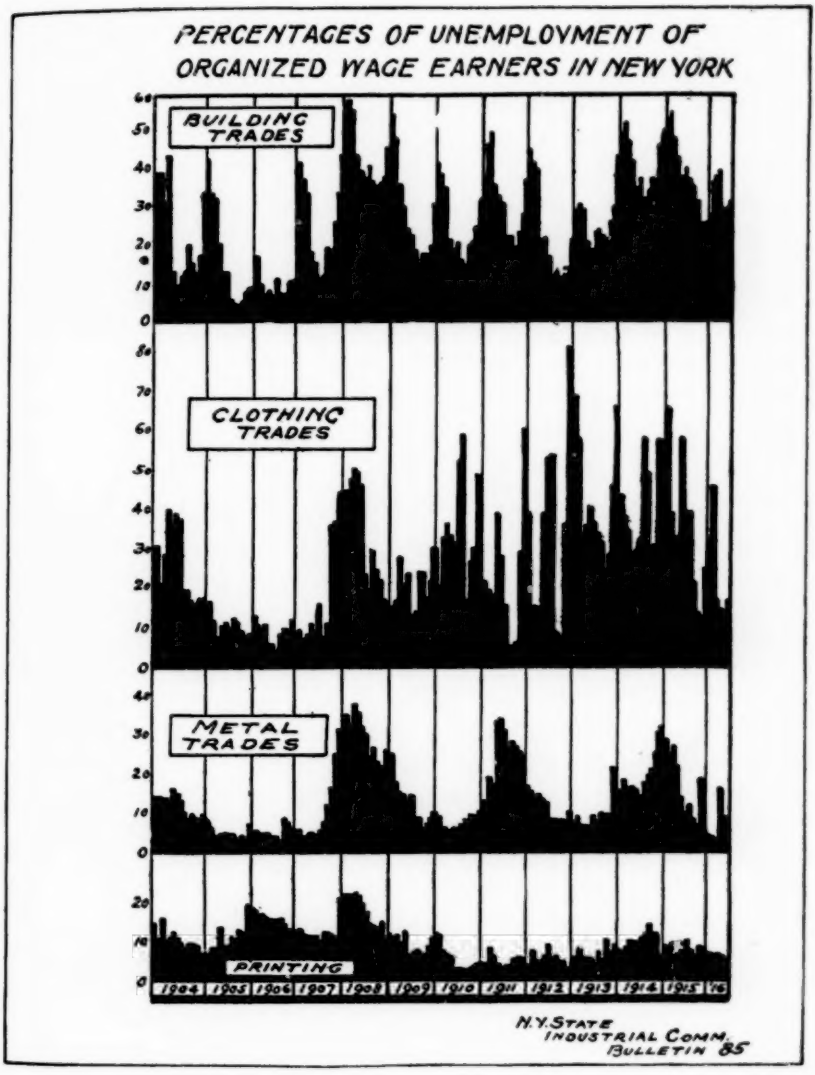
Data on unemployment are sorely needed at present, but the number of items to be secured and the labor of preparing them should both be reduced to a minimum. It is better, however, to get four items that are really needed for information than three which do not tell us what really ought to be known.

Better methods of measuring unemployment can also be worked out



for particular industries. This should be seriously undertaken by industrial and business statisticians.

The three accompanying charts are based on data supplied by state bureaus. The first chart, showing the fluctuations in the unemployment of organized wage earners in Massachusetts, is based on data


PERCENTAGES OF UNEMPLOYMENT OF ORGANIZED WAGE EARNERS IN NEW YORK
BUILDING TRADES
CLOTHING TRADES
METAL TRADES
PRINTING
1904 1905 1906 1907 1908 1909 1910 1911 1912 1913 1914 1915 '16
N.Y. STATE INDUSTRIAL COMM. BULLETIN 85

and a chart furnished by Mr. Roswell F. Phelps, director of the Division of Statistics of the Department of Labor and Industries.

The second chart, showing variations in the number of factory employees in representative factories in New York and Wisconsin, is made from data supplied by Dr. E. B. Patton, chief statistician of the New York Department of Labor, and by the Wisconsin State Industrial Commission. The New York curve starts at 100 in July, 1914;

although 100 in the Wisconsin curve is as of February, 1915, the two curves are plotted in this way in order to show more clearly the similarity of movement. The line of normal increase of factory wage earners in New York State from 1904 to 1914—about 12 per cent for the five year period—is based on the figures of the United States Census of Manufactures.

The third chart, showing seasonal and other fluctuations in employment among organized wage earners in certain trades in New York State, 1904–15, is based on charts published in Bulletin No. 85 of the New York State Industrial Commission.

A METHOD OF ESTIMATING THE POPULATION OF CONTINENTAL UNITED STATES FOR INTERCENSAL YEARS

By Donald R. Belcher*

The United States Department of Commerce publishes annually in its *Statistical Abstract* a table showing the estimated population of the United States by years over a considerable period, based on the returns of the decennial censuses of population. For the years prior to that of the latest census, the numerical increase in population in any year is assumed to be one-tenth of the numerical increase shown for the decade to which that year belongs. For years subsequent to the latest census, the annual increase is assumed to be equivalent to that assumed for the years of the preceding decade. These processes of estimation may be simply described as straight-line interpolation and straight-line extrapolation respectively.

It is probably true that until the beginning of the World War these methods yielded results that were not seriously misleading. But during the war a number of influences were at work, the net result of which was to restrict population growth in the United States. Indeed, straight-line extrapolation as applied to the years between 1910 and 1920 on the basis of the censuses of 1900 and 1910 showed an estimated population for 1920 nearly two millions greater than that actually determined by the 1920 census. And now that the 1920 returns are available, straight-line interpolation is equally unsatisfactory for determining the intercensal population during that decade, since it is apparent that these restrictive influences did not become operative until 1914.

An analysis of available statistics on births, deaths, immigration, and emigration during the period from 1910 to 1920 has been found to

* Mr. Belcher is associated with the Statistical Division, American Telephone and Telegraph Company.

provide a fairly satisfactory basis for estimating the population growth for the intervening years. The purpose of this paper is to present the results of that analysis and to suggest a method which can be applied to the estimation of growth during the period prior to the taking of the 1930 census.

The population of continental United States as shown by the last three census reports was as follows:

June 1, 1900	75,994,575
April 15, 1910	91,972,266
January 1, 1920	105,710,620

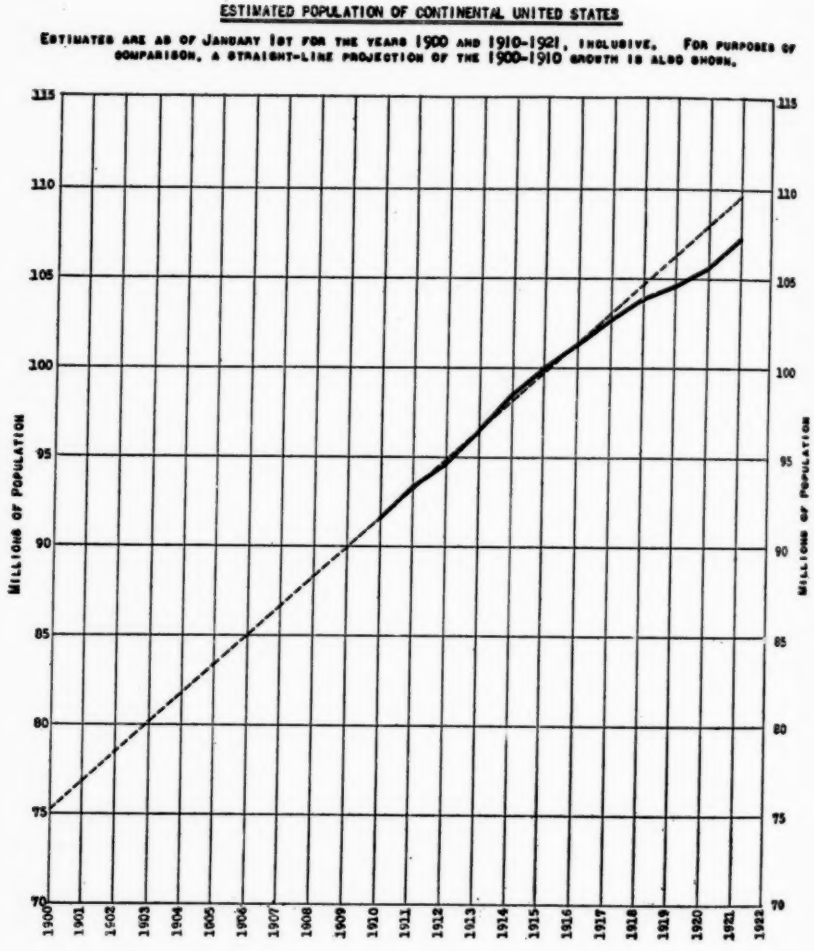
In order to make this study as of January 1 of each year, the 1910 census figure was moved back three and one-half months by the assumption of a straight-line growth between the June 1, 1900, and the April 15, 1910, censuses. The result is an estimated figure for January 1, 1910, of 91,500,350. The growth in population from this date to January 1, 1920, is then 14,210,270, or an average of 1,415,109 a year.

Data on immigration and emigration are available by months since July, 1907. The annual increase in population due to excess of immigration over emigration was computed throughout the period and is shown in Column 1 of Table I.

TABLE I

Calendar year	Column 1 Actual excess of immigration over emigration	Column 2 Estimated deaths in United States	Column 3 Estimated births in United States	Column 4 Net annual increase in population
1910	763,918	1,267,101	2,281,904	1,778,721
1911	326,041	1,302,498	2,325,953	1,349,496
1912	646,742	1,308,227	2,369,106	1,707,621
1913	1,017,957	1,254,036	2,413,496	2,177,417
1914	262,853	1,289,339	2,459,644	1,433,158
1915	43,005	1,293,987	2,506,196	1,255,214
1916	263,887	1,422,182	2,534,859	1,376,564
1917	80,702	1,452,152	2,588,486	1,217,036
1918	41,471	1,888,681	2,577,299	730,089
1919	4,636	1,377,331	2,401,140	1,028,445
Total	3,451,212	13,855,534	24,458,083	14,053,761

Data on the number of deaths annually in the registration states are available since 1900. Although the number of states reporting has varied considerably, it was possible to select twenty-one identical states for which data are available throughout the decade, 1910–20. The population of these states in 1910 was 53.4 per cent and in 1920, 54.6 per cent of the population of continental United States. The corresponding ratio for each of the intervening years was estimated by



means of a straight-line interpolation between 53.4 per cent and 54.6 per cent. On the basis of these ratios and the actual number of deaths in the twenty-one states, the total number of deaths in the United States was estimated for each of these years (Column 2, Table I). It is true that the Bureau of the Census has published annual death-rates for the registration states. But these rates are based on population estimates secured by straight-line extrapolation since 1910 and are, therefore, considerably too low. For this reason it seems preferable to use the actual number of deaths.

The determination of the number of births offers additional difficulties, since the only data available are the number of births in each of

the states for the year ending June 1, 1900, and in certain registration states by calendar years beginning with 1915. Comparison of the figures for ten identical states showed that although the number of births decreased decidedly in 1918 and in 1919, the figures for the years 1900, 1915, 1916, and 1917 were fairly consistent with a constant rate of increase. The number of births was therefore estimated for the years 1910–14 inclusive by means of this constant rate. These figures, however, are for ten states, the total population of which in 1910 was 30.68 per cent and in 1920, 30.71 per cent of that of the United States. These and interpolated ratios were then used to compute the estimated number of births in the United States (Column 3, Table I).

The annual population increases due to the net results of immigration, emigration, estimated deaths, and estimated births are shown in Column 4. The total of these increases is, however, slightly smaller than the actual population increase from 1910 to 1920. In other words, this total added to the 1910 census figure gives an estimated population for January 1, 1920, of 105,554,111, or 0.15 per cent lower than that shown by the census. On the other hand, the straight-line projection method, based solely on the 1900 and 1910 censuses, gives an overestimate of 1.86 per cent, or a percentage error more than ten times as great. Furthermore, straight-line interpolation between 1910 and 1920 is unsatisfactory because, as already pointed out, it fails to take into consideration the restrictive influences in effect during the latter half of the decade.

Estimates for the years between 1910 and 1920 have, therefore, been obtained from the increases shown in Column 4 by spreading the small cumulative error over the ten years. Table II shows for each year the resulting population estimate as of January 1, and an estimated growth during the year.

TABLE II

Estimated population of continental United States

Calendar year	Population January 1	Growth during the year
1910	91,500,350	1,781,567
1911	93,281,917	1,355,186
1912	94,637,103	1,716,158
1913	96,353,261	2,188,798
1914	98,542,059	1,447,387
1915	99,989,446	1,272,288
1916	101,261,734	1,396,484
1917	102,658,218	1,239,801
1918	103,898,019	755,700
1919	104,653,719	1,056,901
1920	105,710,620	1,609,658
1921	107,320,278	.....

As a matter of interest, the growth during 1920 has been estimated on the basis of actual immigration and emigration data and projected births and deaths in the registration states. This results, as shown also in Table II, in an estimated population of 107,320,278 for January 1, 1921. It is true, of course, that in applying this method to projections of future growth no information as to the trend of the ratio between the population of the selected registration states and that of the United States is available. The error involved in projecting the ratios found in the 1910–20 decade will probably not be appreciable.

The accompanying chart shows the population estimates for the years 1910 to 1921 inclusive, and a straight-line projection of the 1900–10 growth.

A SIMPLE GRAPHIC CONSTRUCTION FOR FARR'S RELATION BETWEEN BIRTH-RATE, DEATH-RATE, AND MEAN LENGTH OF LIFE*

By Alfred J. Lotka

In a previous article† the writer has discussed the rational basis of Farr's rule‡

$$\frac{1}{3}\cdot\frac{1}{b}+\frac{2}{3}\cdot\frac{1}{d}=L \tag{1}$$

where

$$b = \text{birth-rate per head} \tag{2}$$
$$d = \text{death-rate per head} \tag{3}$$
$$L = \text{mean length of life.} \tag{4}$$

Writing Farr's rule in the more general form

$$P\frac{1}{b}+Q\frac{1}{d}=L \tag{5}$$

or

$$p\frac{1}{b}+q\frac{1}{d}=1 \tag{6}$$

$$(b-p)(d-q)=pq \tag{7}$$

it was shown in the paper cited how the values of P, Q (represented in Farr's formula by the approximate numerical coefficients $\frac{1}{3}$ and $\frac{2}{3}$) could be determined exactly.

* *Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health Johns Hopkins University.* No. 46.

† *Quart. Pub. Am. Statis. Assn.*, Sept., 1918, p. 121.

‡ Quoted by Newsholme, *Vital Statistics*, 1899, p. 301.

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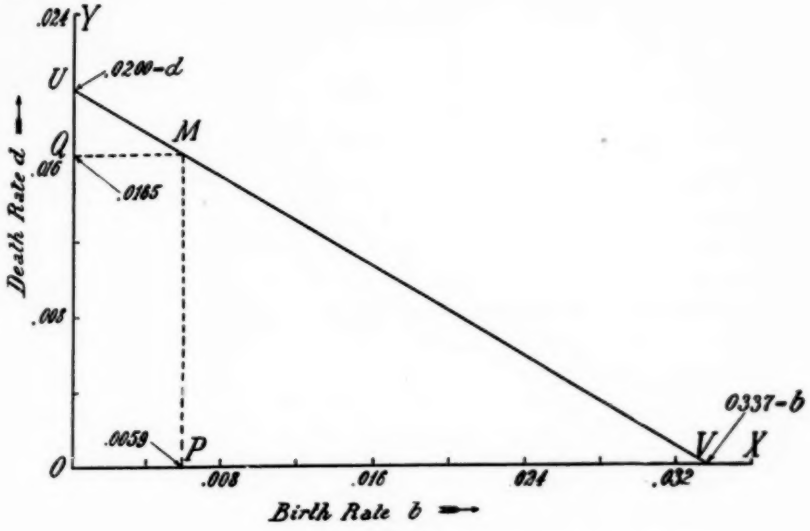

FIG. 1.

Along OX mark off a length $OP = p = \frac{P}{L}$ (8)

Along OY mark off a length $OQ = q = \frac{Q}{L}$ (9)

Complete the rectangle $QOPM$
Suppose we are now given

$$b = 0.0337 \tag{10}$$

It is required to find d
Along OX mark off $OV = 0.0337$ (11)
Join VM and produce it to meet OY in U
Read off at U on the scale of OY

$$d = 0.0200. \tag{12}$$

The figures employed in this example are those for the female population of England and Wales 1871–80, a period during which (as the writer has previously shown) the observed figures for many characteristics of the population agree very closely with those computed for a "Natural Population Norm."* In the particular instance here chosen

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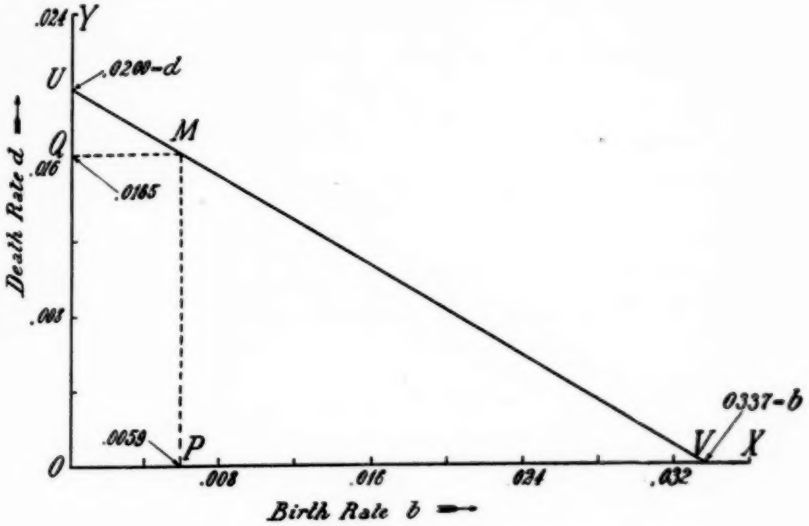

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the numerical values were

$$P = 0.2631 \tag{13}$$

$$Q = 0.7369 \tag{14}$$

$$L = 44.62 \tag{15}$$

$$\frac{P}{L} = 0.01651 \tag{16}$$

$$\frac{Q}{L} = 0.00590 \tag{17}$$

i. e.,

$$0.2631\,\frac{1}{b} + 0.7369\,\frac{1}{d} = 44.62. \tag{18}$$

Putting $b = 0.03373$ (observed) we find (19)

$$d = 0.02000 \text{ (calculated)} \tag{20}$$

as against

$$d = 0.020001 \text{ (observed)}. \tag{21}$$

I take this opportunity thus to supplement the data given for the male population in my previous paper. At the time it was written the extraordinarily close agreement between calculated and observed figures for the female population had escaped my notice.

A VECTOR METHOD FOR COMPUTING A WEIGHTED AVERAGE

By R. von Huhn

In terms of dynamics the arithmetic mean is the center of gravity of a distribution. To illustrate this fact we may assume that certain values represent unit weights which are suspended at intervals upon a massless rod. The ideal rod which is under the action of these weights or external forces is at rest or in static equilibrium when the forces balance each other. The location of the fulcrum is therefore the position of the average with respect to the weights.

As the weights or forces have both magnitude and direction (parallel in this case), the two essentials that are necessary for the treatment of data by vector analysis are given.

Suppose we have at hand the statistical data as shown in the following table:

Daily wage class (unit) A	The number of times each unit is encountered or weights B		Product of weight×unit C
\$1	..	..	..
\$2	4	R_1	8
\$3	9	R_2	27
\$4	3	R_3	12
\$5	..	..	..
\$6	5	R_4	30
	21		77
77 : 21 \$3.66 weighted average			

It is possible to arrive at the value \$3.66 by applying the vector method and especially the funicular polygon. This brings out in a striking way the close relationship of certain dynamic principles to certain phases of the statistical science.

Column A in the table represents the unit scale from \$1–\$6. This scale is drawn in Figure 1 as MN, each unit representing one dollar =

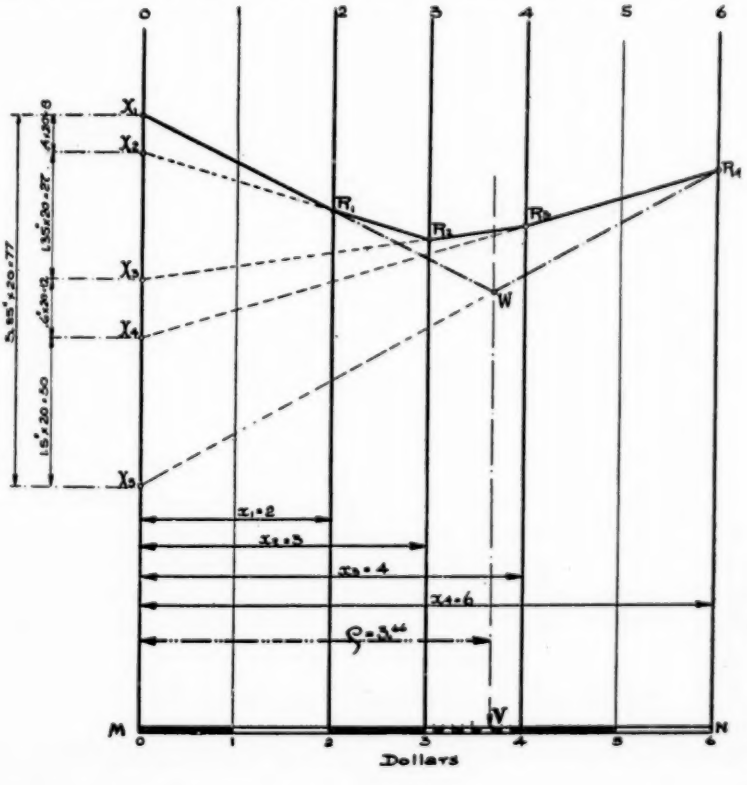

Fig. 1.

one inch. Seven perpendicular lines are drawn in each point; in 0, 1, 2, 3, 4, 5 and finally 6.

The next step is shown in Figure 2, the Polediagram. Draw a line AE parallel R_4N and choose a scale for the weights of Column B in the table. The scale chosen in Figure 2 is $1'' = 5$ dollars. Then add the vectors of the parallel forces $R_1\ R_2\ R_3\ R_4$ graphically.

Then we have:

$$\begin{aligned} AB &= R_1 = 4 \\ BC &= R_2 = 9 \\ CD &= R_3 = 3 \\ DE &= R_4 = 5 \\ \hline AE &= \Sigma R = 21 \end{aligned}$$

These are the weights which have their corresponding distances from Point O such as x_1, x_2, x_3, x_4, the product of each weight and its cor-

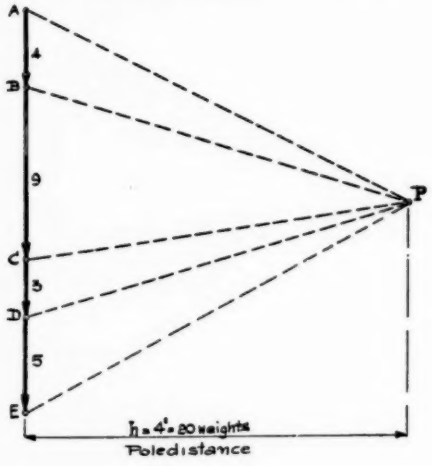

Fig 2.

responding distance giving the moments in the system. Then choose a pole distance $h = 4$ inches (each inch representing 5 weights) perpendicular from the vectors. The pole distance h may be made any length desired as long as a certain scale is chosen. If Point P is connected with the points A, B, C, D, and E the polediagram is completed.

Then we proceed as follows:

At an arbitrary point X draw a parallel line XR_1 to AP until it intersects with the perpendicular line $2-2$ in R_1; through R_1 draw a parallel line R_1R_2 to BP until it intersects with the perpendicular line $3-3$ in R_2; through R_2 draw a parallel line R_2R_3 to CP until it inter-

sects with the perpendicular line 4−4 in R_3; through R_3 draw a parallel line R_3R_4 to DP until it intersects with the perpendicular line 6−6 in R_4.

Then draw a line R_4W through R_4 parallel to EP and extend the line X_1R_1 until it intersects with R_4W in W.

If we draw a line through W parallel to R_4N the point V will give the location of the weighted average on the unit scale MN and MV = 3.66 inches = \$3.66.

Proof:

Extend R_4W until it intersects in X_5 with the perpendicular line oo
R_4R_3 " " X_4 "
R_3R_2 " " X_3 "
R_2R_1 " " X_2 "

Then $\triangle PAB$ is similar $\triangle R_1X_1X_2$

$$\therefore AB : h = X_1X_2 : x_1$$

hence $$AB \cdot x_1 = h \cdot X_1X_2$$

again $\triangle PBC$, $\triangle PCD$, $\triangle PDE$ are similar to $\triangle R_2X_2X_3$, $\triangle R_3X_3X_4$, $\triangle R_4X_4X_5$ respectively.

Furthermore:

$$AB \cdot x_1 = h \cdot X_1X_2 = .40'' \times 20 = 8.^\circ$$
$$BC \cdot x_2 = h \cdot X_2X_3 = 1.35'' \times 20 = 27.^\circ$$
$$CD \cdot x_3 = h \cdot X_3X_4 = .60'' \times 20 = 12.^\circ$$
$$DE \cdot x_4 = h \cdot X_4X_5 = 1.50'' \times 20 = 30.^\circ$$

Added $$h(X_1X_2 + X_2X_3 + X_3X_4 + X_4X_5) = 3.85'' \times 20 = 70.^\circ$$

h = 20 weights = Poledistance

$$X_1X_5 = 3.85''$$

Furthermore:

$$\triangle APE \text{ similar } \triangle WX_1X_5$$
$$\therefore \triangle APE : \triangle WX_1X_5 = h : \rho$$

ρ = distance MV = Location of weighted average

hence

$$AE : X_1X_5 = h : \rho$$
$$\rho = h\frac{X_1X_5}{AE}$$
$$= 20\frac{3.85}{21} = \frac{77}{21}$$
$$\rho = 3.66 \text{ Dollars.}$$

REPORT UPON THE APPORTIONMENT OF REPRESENTATIVES*

In accordance with the request of the Senate Committee on the Census, transmitted by your letter of February 18, 1921, our committee has examined the proposed methods of apportioning representatives among the several states. We have had before us documents and correspondence furnished by the Director of the Census, by Dr. J. A. Hill of the Bureau of the Census, and by Professors W. F. Willcox and F. W. Owens of Cornell University and E. V. Huntington of Harvard University. Professor Willcox is a member of our committee, but, because he devised one of the methods considered, he does not join in this report. Professors Huntington and Owens, upon our invitation, appeared before the committee and explained the methods they had devised. We have also sought the counsel of Professor Irving Fisher of Yale University. Our conclusions may be summarized as follows:

1. The "method of equal proportions" leads to an apportionment in which the ratios between the representation and the population of the several states are as nearly alike as is possible. It thus complies with the conditions imposed by a literal interpretation of the requirements of the Constitution.

2. The "method of major fractions" has back of it the weight of precedent. Logically, however, it can be supported only by holding that the Constitution requires, not that the ratios between the representation and the population of the several states shall be equal, as nearly as is possible, but that the representation accorded to individuals or to equal groups of individuals in the population (that is, their "shares" in their respective representatives) shall be as nearly uniform as is possible, irrespective of their places of residence.

3. It is not clear that the special interpretation of the Constitution which alone is consistent with the use of the "method of major fractions" is to be preferred to other possible special interpretations which lead to other methods of apportionment. We conclude, therefore, that the "method of equal proportions," consistent as it is with the literal meaning of the words of the Constitution, is logically superior to the "method of major fractions."

In the sections which follow we explain the considerations which have led us to these conclusions.

* This report was made by the Joint Committee of the American Statistical Association and the American Economic Association to Advise the Director of the Census in the form of a letter to United States Senator Howard Sutherland, chairman of the Senate Committee on the Census.

I. THE ALTERNATIVE METHODS

The two methods of apportioning representatives with respect to which the judgment of the committee is sought are, we understand, first, the "method of major fractions," devised by Professor W. F. Willcox in 1910 and used in the apportionment of 1911,* and, second, the "method of equal proportions," devised by Professor E. V. Huntington in December, 1920, and brought to the attention of Congress through a letter to Chairman Siegel of the House Committee on the Census.† To the "method of major fractions" there may be joined a method devised by Professor F. W. Owens in February, 1921, and named by him the "method of least errors." Professor Owens' method, it happens, although independently conceived and resting on somewhat different premises, leads to precisely the same apportionment as the "method of major fractions." It may be taken, therefore, as adding strength to the claims of that method. Moreover, Professor Huntington's "method of equal proportions" is in effect a modified form of the "method of alternate ratios," devised by Dr. J. A. Hill of the Bureau of the Census and recommended by Dr. E. Dana Durand, then Director of the Census, for adoption in 1911.‡ In discussing the "method of major fractions" we have taken into account, therefore, the suggestions of Professor Owens as well as those of Professor Willcox, and similarly we have joined Dr. Hill's suggestions to those of Professor Huntington in our consideration of the "method of equal proportions."

II. FACTORS THAT INFLUENCE A CHOICE

Judged by purely technical criteria, the method of equal proportions is superior to the method of major fractions. By "purely technical criteria" we do not mean matters of mathematical detail. Both methods are, in such respects, mathematically sound. The difference between them is one of logic rather than mathematics, of premises rather than of methods of deduction. Both methods are logically defensible. The choice between them is not a choice between the right and a wrong method. The question is a more difficult one: which of two defensible methods is the better?

We say that we base our judgment on "purely technical criteria" in order to make it clear that we have left out of account, as beyond our province and our competence, certain matters which Congress perhaps may deem important. In particular we have given no weight either to precedent and tradition or to subtleties of interpretation, such as

* Cf. Report of the House Committee on the Census, 1911, 82d Congress, 1st Session, House Report No. 12, pp. 1–42.

† Congressional Record, January 19, 1921, p. 1791.

‡ Report of the House Committee on the Census, 1911, pp. 43–108.

might find in the language of the Constitution, or in the circumstances of its history, an indication that some one particular significance or meaning should be attached to apportionment ratios.

If any weight is to be given to precedent, it will count for the method of major fractions. Professor Willcox, who devised the method used in the apportionment of 1910, has explained that his aim was "simply to provide a set of tables which would conform to the desire and purpose of Congress," as expressed in previous reports, debates, and votes upon apportionment. In the past Congress has, in fact, indicated its belief that if a state's population exceeds any exact multiple of the population of the average congressional district by a "major fraction," the major fraction gives the state a prima facie claim to another representative. The method used in the apportionments of 1840 and 1910, and now called the method of major fractions, satisfies the requirements a recognition of such claims makes as closely as is practicable.

What weight should be attached to precedent is, of course, for Congress to determine. While, on the one hand, it might be urged that if any other method is to displace the method of major fractions the other method must have a very considerable margin of superiority, it might possibly be maintained, on the other hand, that in view of the inherent complexities of the problem and the likelihood that all of its different aspects have not been presented to Congress in the past, it would be wrong to consider the case as in any degree closed. At any rate, we venture to suggest, there is one factor whose importance has increased and is likely to increase. In the past the increased representation the more rapidly growing states have gained has seldom been at the direct expense of other states. With the interests of each state thus, in a measure, safeguarded, some of the mathematical niceties of exact apportionment might be disregarded. But once the size of the House of Representatives is definitely restricted, the problem of apportionment takes on new importance. It is no longer merely a question of what states shall gain representatives. It includes also the more awkward question of what states shall lose representatives. It is possible that under such conditions Congress may hold that it is less important that the method of apportionment be based on precedent than that it should rest on the best possible logical and mathematical foundation, so as to be immune, so far as may be, from successful criticism and attack.

III THE CONSTITUTIONAL REQUIREMENT

We do not understand that the constitutionality of either of the proposed methods is now in question, nor do we assume that the con-

stitutionality of either of them is likely to be challenged. Both methods comply with the requirements of the Constitution in that each leads to an apportionment of representatives "among the several states according to their respective numbers." The problem merely is, which method leads to a more nearly *exact* apportionment?

The words of the Constitution are: "Representatives shall be apportioned among the several states according to their respective numbers, counting the whole number of persons in each state, excluding Indians not taxed." (Fourteenth Amendment, Sec. 2.) As we have said, we do not conceive our task to be that of constitutional interpretation. Nevertheless it is impossible to come to any conclusion respecting the relative merits of the competing methods of apportionment without careful consideration of their relation to the requirements of the Constitution. For this purpose we have taken the words of the Constitution at their face value. We have tried to avoid reading into them any implications not warranted by a rigidly literal interpretation.

For example, it has been held that these words imply that a group of individuals resident in any part of the United States is entitled to the same amount or share of representation in Congress as any other group of individuals of equal numbers resident in any other part of the United States. Or it might be held that it is implied that the population of the different congressional districts should be as nearly equal as possible. These two implications, seemingly somewhat alike, lead logically to two very different methods of apportionment. Congress might draw inferences like these from the stipulations of the Constitution. Our committee has not ventured to go that far. For a similar reason we have given no weight to the suggestion that the equal representation of the states in the Senate indicates that preference should be given to that method of apportioning representatives in the House which is more favorable to the larger states. That is clearly a matter of congressional policy, outside the field of inquiry assigned to our committee.

Confining ourselves, then, to a literal reading of the constitutional clause, we find in it two guiding principles that have an important bearing upon the problem in hand.

First, representatives are to be "apportioned among the several states." We have not thought it advisable for us to go beyond this clear stipulation by treating the problem as one of adjusting the equities of individual citizens, of groups of individuals, or of their representatives. As we have suggested, the restatement of the problem in some such manner might point to a different solution from the one we favor. In its original form (Constitution, Article I, Sec. 2), the clause in question, covering as it did the apportionment of both representa-

tives and direct taxes, was born of an effort to adjust the conflicting interests of the different states. We have not assumed that its purpose and meaning have changed.

Second, representatives are to be apportioned among the states *according to their respective numbers.* Here we have a plain stipulation that the allocation of representatives among the states is to be determined on the basis of *proportions* or *ratios.* The ratio between the representation assigned any one state and the representation assigned any other state is to accord with the ratio between the population of the one and the population of the other. But there is no stipulation with respect to the precise way in which such proportions are to be expressed or stated.

IV. THE MEASUREMENT OF THE EQUALITY OF RATIOS

No method of apportionment, of course, can assign representatives to the several states in *exact* proportion to their respective numbers. The problem then is, in Daniel Webster's phrase, to reach a result that shall be "as near as may be" to an exact apportionment. The principal difficulty occurs just at this point. How is "nearness" to an exact apportionment to be measured? How is the difference between two states with respect to the relation between their representation and their numbers to be determined? These questions are included, in part at least, in a larger problem—how is the "nearness" of any two numbers to be determined?

The "nearness" of two numbers may be measured either *absolutely* or *relatively.* For example, which are nearer, the numbers 3 and 5, or the numbers 45 and 50? The absolute difference between 3 and 5 is only 2, while between 45 and 50 it is 5, so that by this test 3 and 5 are "nearer" than 45 and 50. But while 3 is 60 per cent of 5, 45 is 90 per cent of 50, so that *relatively* 45 and 50 are "nearer" than 3 and 5. Suppose that state A is given 20 representatives, its exact quota being 20.6, while state B is given 2 representatives, its exact quota being 2.4. Which state is nearer to its exact quota? Absolute nearness is closer in the case of state B, since 0.4 is smaller than 0.6; but relative nearness is closer in the case of state A, since state A has 97 per cent of its exact quota, while state B has only 83 per cent. Expressed in this manner, the measurement of "nearness" by absolute differences leads to the method of major fractions, while its measurement in relative or percentage terms leads to the method of equal proportions.

For the general problem of the measurement of the "nearness" of two numbers, there is no single exclusive solution. The appropriate method must be determined by the nature of the concrete problem in

hand. For example, temperatures of 5 degrees and 10 degrees above zero are "nearer" than are temperatures of 90 degrees and 100 degrees. But a firm with assets of $50,000 and liabilities of $100,000 is further from solvency than a firm with assets of $900,000 and liabilities of $1,000,000.

Although the problem of the best method of measuring the "nearness" of *numbers* is thus, as a general problem, ambiguous, such is not the case with respect to the "nearness" of *ratios* or proportions. A ratio between numbers is not itself a quantity, yet there is no way of measuring the absolute difference between ratios, except by putting them into the form of fractions and treating them as numbers or quantities. In order that a ratio may be treated as a fractional quantity, certain assumptions must be made which may not, in a particular case, be warranted. But ratios may properly be compared, *as ratios*, with respect to their *relative* "nearness." Thus to say that the ratio between 2 and 4 is 83⅓ per cent of the ratio between 3 and 5, is merely to say that the first ratio is related to the second ratio as the ratio between 83⅓ and 100 is related to the ratio between 100 and 100.

The problem in hand, or rather so much of that problem as we take to be within the field of our inquiry, is to determine which of the methods under review will give an apportionment of representatives that will be, "as near as may be," in accordance with the populations of the several states. This is a matter of making ratios or proportions as nearly alike as possible. For the reasons we have outlined the alikeness or nearness of ratios is best measured in relative terms. This is one ground on which technical superiority must be assigned to the method of equal proportions.

V. DIFFERENT WAYS OF EXPRESSING RATIOS

When apportionment ratios are expressed as fractions, further difficulties appear. Such fractions may take several forms. For example, their numerators may express the number of representatives to be assigned the several states, and the denominators their populations. Or population may form the numerator and the number of representatives the denominator. Or, again, the representation of any two states may be compared by means of a pair of fractions. One fraction has for its numerator the population of one state and for its denominator the population of the other. The other fraction has for its numerator the number of representatives assigned the first state, and for its denominator the number of representatives assigned the other.* Which among these different sets of fractions is to be selected?

* Denoting the number of representatives assigned to states A and B by a and b, and their respective populations by A and B, there are the following possible classes of fractions: (1) a/A, b/B, etc.; (2) A/a, B/b, etc.; (3) A/B, a/b, etc.; (4) B/A, b/a, etc.

If the method of equal proportions is used, the choice is a matter of indifference. Any or all of the possible forms of fractions will lead to the same apportionment. That is, the apportionment which will bring one set of fractions "as near as may be" to equality, will also minimize the differences in the other sets of fractions. This is because when the "nearness" of ratios is measured on a relative scale, the results do not depend upon the particular form of the fractions by means of which the ratios are expressed.

When, however, the "nearness" of fractions is measured by their absolute differences, so that the fractions are treated as quantities rather than as merely expressing ratios, the form of the fraction used in the comparison becomes a matter of importance. The measurement of "nearness" by means of absolute differences does not of itself lead to the method of major fractions. That method requires the further assumption that the significant fractions are those in which the numerators denote representatives and the denominators population. Inverted, so that their numerators denote population, and with their "nearness" measured by their absolute differences, these fractions lead to a very different scheme of apportionment, which, on mathematical (though not on historical) grounds has an equally valid claim to the name, method of major fractions. The method of equal proportions, it is interesting to note, gives results that are intermediate between those given by these two ways of measuring "nearness" by absolute differences. It is somewhat more favorable to the small states than is the method of major fractions and somewhat more favorable to the large states than that method would be if it made use of the other form of fractions.

On purely mathematical grounds there is no reason to prefer one of these two forms of fractions to the other. The two are equally valid as expressions of apportionment ratios. It thus appears that unless other than mathematical reasons can be found for preferring one form of fraction, the method of major fractions encounters the formidable objection that it leads to results that are self-contradictory.

If preference is to be given to one form of fraction, it can only be because different *meanings* are attributed to the different forms. Now as long as these fractions are construed merely as expressing ratios, there can be no difference in their meanings. To state that the ratio of representatives to population is as 1 to 240,000 is to state that the ratio of population to representatives is as 240,000 to 1. A familiar example may make the matter clear. The speed of a railway train or of an aeroplane is stated in terms of miles per hour; the speed of a runner in terms of minutes per mile. The meaning is the

same in both cases: the ratio between the two magnitudes: distance traversed and time elapsed. Science, it is true, has adopted the convention of expressing the time-distance ratio as a fraction in which distance is the numerator and time the denominator. But this is merely a matter of convenience. In inverted form the ratio, *as a ratio*, would have the same *meaning*.

If distinctive meanings, therefore, are to be attached to the different forms of fractions which express apportionment ratios, they must be construed to be something more than mere ratios. This construction may be accomplished by using common denominators or by reducing the denominators to unity, so that the numerators become the quotients of the original numerators divided by the original denominators. To the numerators, taken by themselves, concrete meanings can be attributed. Thus, the number of representatives assigned to a state divided by a number representing its population gives a number which may be interpreted as the "average representation accorded to an individual in the state's population." Similarly, the quotient obtained by dividing the state's population by a number corresponding to the representatives assigned it, may be construed as "the average size of a representative's constituency," or "the average size of a congressional district within the state."

So far as mathematical considerations are concerned, the processes by which one meaning or the other is imputed to apportionment ratios are wholly arbitrary. If the method of major fractions is to be supported, it must be by holding that what the Constitution requires is, after all, not that the ratios of representatives and population shall be equal "as near as may be" for the several states, or even that the average size of congressional districts shall be as equal as possible as among the several states, but that the differences in "the representation accorded an individual" shall be minimized. The absence of any clear reasons (save those of precedent) for emphasizing the "representation accorded an individual" rather than "the size of an average congressional district," or either of them rather than that simple proportioning of representatives according to population which the constitutional clause, taken at its face value, seems to require, establishes the second ground on which the method of equal proportions is to be preferred to the method of major fractions.

VI. THE USE OF SQUARES IN THE METHOD OF LEAST ERRORS

Professor Owens has shown that the apportionment which will make the sum of the squares of the errors (*i. e.*, the differences between the actual share a citizen of a state has in his representative and the exact

share to which he is "entitled") a minimum will be the same apportionment as would be reached by the method of major fractions. This result affords additional evidence that results which the method of major fractions gives are logical deductions from the premises of that method, and possibly makes the logical premises of that method somewhat clearer. But this use of the method of least squares does not strengthen the claims of the method of major fractions as against the method of equal proportions. The "errors" whose squares Professor Owens seeks to minimize correspond to those "absolute differences" which are less appropriate measures of the "nearness" of apportionment ratios than are the relative differences utilized in the method of equal proportions. Moreover, the method of least squares is applicable to any of the forms of fractions which may be used to express apportionment ratios, and thus leads to self-contradictory results. In short, as used by Professor Owens it not only presupposes the validity of absolute differences, as a measure of "nearness," but it also requires, as Professor Owens himself suggests, that only one of the possible types of fractional expression of the apportionment ratios shall be utilized.

VII. SUMMARY OF CONTROLLING CONSIDERATIONS

1. It is clear that the Constitution requires that the allocation of representatives among the several states shall be proportionate to the distribution of population. It is not equally clear that there is anything in the constitutional requirement which suggests that one of the forms in which such apportionment ratios or proportions may be expressed should be preferred to another.

2. The method of major fractions utilizes only one of several ways of expressing apportionment ratios. The method of equal proportions utilizes all of these ways without inconsistency. The latter method, therefore, has a broader basis.

3. There is no mathematical or logical ground for preferring the one form of expression of the apportionment ratio used in the method of major fractions to other forms of expression. These other forms lead, when similar processses of computation are employed, to different and therefore inconsistent results.

4. The method of major fractions logically implies preference for a special meaning which may be attached to one of the forms in which apportionment ratios may be expressed. To attach to ratios meanings which vary with the forms in which the ratios are expressed is to interpret them as something else than ratios.

5. In the method of major fractions the "nearness" of the ratios

of representatives and population for the several states is measured by absolute differences. The method of equal proportions utilizes relative differences. The relative scale is to be preferred.

Respectfully submitted

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A NATIONAL INDEX OF RETAIL TRADE

The Federal Reserve system is at present receiving monthly reports from about 350 department stores, located in all sections of the United States. The statistics obtained from these stores are as follows: percentage change of net sales from sales in corresponding month of previous year, percentage change of stocks from corresponding month of previous year, percentage change of stocks from previous month, and ratio of outstanding orders at close of month to total purchases in previous year.

Indices for these items have been published for each of the twelve Federal Reserve districts since August, 1920, but no national index was made previous to August, 1921. At that time a national index was constructed by combining weighted percentages for the several districts. These weights were secured by finding the percentage distribution of population and of banking resources, respectively, in 1920, among the Federal Reserve districts. A weight is obtained for the retail trade of each district in its relation to the retail trade of the nation by multiplying the population percentage of the district by two and adding the district's percentage of total banking resources.

The monthly comparisons of changes in retail sales and stocks since August, 1920, for the United States are shown in the following table:

TABLE I
CHANGES IN VOLUME OF SALES AND OF STOCKS IN COMPARISON WITH CORRESPONDING MONTH IN PREVIOUS YEAR

	Sales Per cent	Stocks Per cent
1920		
August	+21.2	+37.0
September	+13.6	+28.2
October	+10.0	+26.4
November	+13.4	+16.7
December	+ 3.1	+ 6.2
1921		
January	− 4.7	− 9.4
February	+ 0.8	−14.4
March	− 3.0	−17.9
April	− 4.2	−18.9
May	− 8.6	−17.7
June	−10.7	−17.4
July	−15.1	−19.1
August	−11.9	−19.0

There seems to be a moderate degree of direct correlation between these two series, but it must be remembered that the figures are comparisons with the corresponding month in the previous year, and are consequently not a sensitive measure of changes from month to month.

As monthly changes in size of stocks are available, it is possible to construct an index of stocks for the entire period on a fixed base. In Table II this index of stocks is compared with the ratio of outstanding orders to purchases in the previous calendar year (it should be noted that the base of this second series changes in January, 1921):

TABLE II

COMPARISON OF STOCKS WITH OUTSTANDING ORDERS

Close of	Value of stocks (July, 1920=100)	Ratio of outstanding orders to purchases in previous calendar year
1920		
August	107.8	15.9
September	116.0	11.9
October	117.1	7.1
November	111.2	5.5
December	87.0	5.1
1921		
January	80.6	7.0
February	84.7	8.1
March	90.4	6.2
April	92.1	6.0
May	89.9	5.5
June	86.0	6.4
July	84.4	8.4
August	90.0	8.3

These figures indicate that orders tend to increase when stocks are declining, and to decrease when stocks are accumulating, but the data are insufficient to furnish definite conclusions concerning the average lag of this inverse correlation.

In conclusion it should be stated that national indices of retail trade are as yet in only a preliminary form. As soon as data are available from all sections of the country in dollar amounts instead of percentages, it will be possible to construct series of relatives with a uniform base period which will warrant a more extended analysis.

LAWRENCE B. MANN

PROGRESS OF FOURTEENTH CENSUS WORK

Probably few persons other than those connected with the census realize the magnitude of the undertaking and the difficulties of carrying it to completion within the period prescribed by law. It involves the printing and distribution of 25,000,000 schedules of questions; the organization and supervision of a force of over 90,000 enumerators and special agents employed to make a house-to-house canvass of the entire United States—including all the outlying possessions except the Philippines and the Virgin Islands—and to fill out schedules for 107,500,000 people, 6,500,000 farms, 450,000 manufacturing establishments, and 22,000 mining and quarrying enterprises; the examination, checking, and editing of the

schedules when received; the punching of 300,000,000 tabulation cards; the running of the equivalent of over 2,500,000,000 cards through electrical sorting and tabulating machines; the computation of about half a million percentages, averages, and other ratios; the preparation of elaborate manuscript tables; and, finally, the printing and publication of 11 quarto volumes of about a thousand pages each. Despite the size and complexity of the undertaking, and despite the unusual obstacles which impeded the enumeration and delayed the commencement of the tabulation, the stage of progress now reached is such as to afford a substantial basis for the hope and expectation that the Fourteenth Decennial Census work will be completed within the three-year period prescribed by law—ending June 30, 1922—at least to the extent of having the manuscript for the last of the reports in the hands of the printer before the close of the period. The field work and the checking and editing of the schedules, the card punching, and the machine sorting and tabulating have already been completed.

For the first time in the history of the census the agricultural and manufactures data are being completely tabulated by punch cards and electrical tabulating machines. The machinery employed differs essentially from that used on the population work, however. In one case it is necessary to count units, and in the other to add numbers. Therefore, the machines used for the population census are essentially unit-counting machines, while most of those for the agricultural and manufactures work are integrating machines. In the agricultural work 17 sets of punch cards are used, and in the manufactures work 12 sets.

PUBLICATIONS ISSUED OR SOON TO BE ISSUED

Volume I—Number and Distribution of Inhabitants.—This volume, which was issued August 30, 1921, represents a radical departure from the plan of publication of the Thirteenth Census results. It presents all the census statistics as to total population—that is, population not classified as to sex, age, color, etc.—giving the population of each state and of its counties, cities, townships, and other minor civil divisions. The population of cities having more than 5,000 inhabitants is also shown by wards in all cases where the enumerators' returns make this possible.

The bulk of the volume is made up of six detailed tables giving the population of counties, incorporated places, and other minor civil divisions. The urban and rural population of counties is shown separately. The final table gives the population, by minor civil divisions, of the several outlying possessions covered by the census. Preceding the detailed tables is a summary section consisting of 48 summary tables, explanatory text, and maps and diagrams.

Thus this volume presents all the Fourteenth Census population statistics in which the great majority of users of census figures are interested; and it has been published at a much earlier date, relatively to the date of the enumeration, than any of the Thirteenth Census volumes.

Volume III—Composition and Characteristics of the Population, by States.—This volume will consist in the main of the material contained in the several state bulletins on composition and characteristics of the population, and will correspond, as far as the composition and characteristics statistics are concerned,

to Volumes II and III of the Thirteenth Census reports. Those volumes, however, also contained statistics on number and distribution of inhabitants, corresponding in a measure to those presented in Volume I of the Fourteenth Census reports. In addition, Volume III will include a section giving figures for the United States as a whole, in which the plan of presentation of the statistics will be similar in general outline to that followed in the various state sections. This United States section, which will also be published as a separate bulletin, will present all the composition and characteristics statistics desired by most persons who make use of this class of statistics. No such section appeared in the corresponding volumes of the Thirteenth Census reports. The issuance of state bulletins on composition and characteristics of the population was completed November 17, 1921; and Volume III is now in process of printing.

Volume VI—Agriculture: Reports for States with Statistics for Counties.—This volume will correspond to Volumes VI and VII of the Thirteenth Census reports, and will be issued in three parts, one presenting statistics for the North, one for the South, and one for the West. In addition to the state sections, each identical with the corresponding state bulletin, Volume VI—like Volume III, for population—will contain a section relating to the United States as a whole. No such section appeared in the corresponding volumes of the Thirteenth Census reports. The manuscript for this volume is now in the hands of the printer.

Volume VII—Irrigation and Drainage: General Report and Analytical Tables, and Reports for States, with Statistics for Counties.—This volume, as its title indicates, will consist in part of a general report, covering all sections of the country in which irrigation and drainage enterprises are in operation, and in part of a set of reports for individual states, these state reports being identical with the corresponding state bulletins. Volume VII is now in process of printing.

Press Summaries.—At the Fourteenth Census the Bureau has published its detailed statistics—those pertaining to race, age, illiteracy, nativity, occupational classes, farm acreage, crops, wage earners in manufacturing establishments, value of manufactured products, etc.—in the form of press announcements to a greater extent than has been done heretofore, the purpose being to make public all the census statistics at the earliest possible date. First, press summaries pertaining to certain subjects for individual cities are issued—for example, illiteracy in Chicago, school attendance in Philadelphia, and age distribution in Richmond. As soon as the statistics pertaining to a particular subject for an entire state have been compiled, a state summary is issued; and when the compilation is completed for the entire United States, a final summary for the country as a whole is made public. In this way not only the basic census results but also the detailed statistics are being made public through the medium of the press in the localities to which they relate.

SHOULD THE UNDERGRADUATE BE TRAINED IN ELEMENTARY STATISTICAL METHODS?

Industrial history has been introduced into the high school. Elementary economics is now being taught to sophomores in college. Professional training begins during the last two years of the undergraduate course. The student is

taking special courses in applied economics and social science and his background has widened to include a varied assortment of facts which are alleged to describe or measure the phenomena of community life. What was formerly post-graduate or professional study is now under way before the student reaches the more specialized and technical training of his post-graduate course.

Training in method of approach to a problem, acquaintance with the tools needed in his work, familiarity with principles according to which his sources of information may be judged—these are fundamental matters in the education of students of the social sciences, theoretical and applied. Yet these considerations are neglected by many colleges which have shown a rapid development of courses in this field.

Technique and methods in the natural sciences have been well developed and applied in the laboratory by the student beginning the subject. But many students of social science have not mastered a natural science and, therefore, have not acquired the discipline in methods of assembling data and sifting evidence, nor have they acquired the habit of independent and cautious generalization from known facts.

Furthermore, they leave behind them, early in their course and usually with light hearts, the exact methods of their mathematical studies because they do not expect to apply them. *Algebra*, which has aided in solving the unknown from the known, *logarithms*, which have proved effective tools in computation, the *characteristics of lines and curves*, which are utilized today very effectively to describe the phenomena of social and industrial life and to illuminate their explanation—all these are forgotten because the student, as he begins his studies in the social sciences, has not been shown their significance in his work. It is only after he has forgotten and when there is no time to repair the loss that the professional and research worker faces the need for this mathematical equipment. The engineer has no such trouble with his technical training because he understands what is necessary from the beginning.

Admitting, for the sake of argument, that the above statement of the situation is essentially true, why give the social science student statistics as a remedy? This is a fair question, and an answer should and can be given. The training in statistics must not be conceived in any narrow or professional sense, but as a discipline in scientific method. It is concerned with a body of principles and methods developed to guide the student in the collection and treatment of quantitative data relating to groups or aggregates. Laboratory experience with such data places proper emphasis upon the essentials of scientific method, *i. e., a knowledge of facts which have been classified, an appreciation of their sequence, and a judgment as to their relative significance.* Statistics, regarded as a method of investigation, involves exact measurement, a careful record, intelligent grouping, discriminating analysis, and weighing of evidence.

Especially in the social sciences, where factors are complex and human sympathies are keen, it is difficult to free our judgments from the peculiarities of the individual mind. The scientific attitude of mind prompts the student to seek evidence which will appeal to others than himself as true.

The junior or senior in college who is majoring in the social sciences has

acquired a background of interesting facts, many of them expressed in quantitative form. He has concrete evidence of the dynamic character of social and economic life, the constant changes which render his judgments and conclusions relative to time and place. Using this background of facts, it is possible to develop a scientific attitude of mind in the approach to social and economic problems. The training in statistical methods and practice may be made a means to this most desirable end. *Statistics may serve a purpose for the social sciences similar to that which the laboratory serves in the natural sciences.* If offered to the undergraduate at the threshold of his professional training it will gather together and preserve as useful many of the exact methods of mathematics by showing their application to the data in his field of study. In some respects the college senior is in a much better position to study and profit by statistical methods than the present graduate student.

Finally, to read and appreciate the literature in his field, and to consult intelligently the sources of information, many of which are in quantitative form, the student of the social sciences, theoretical and applied, needs some training in statistical methods. Figures may speak for themselves, but not necessarily the truth. The student faces masses of quantitative data in so-called statistical reports, and he requires some principles by which he may judge their credibility and adequacy for his purposes. Quantitative statements do not usually bear on their face the stamp of their credibility. The independent student must be skeptical of his own facts and those produced by others.

If the undergraduate is trained in the elements of statistical method the way will be clear, when he enters a business school or a graduate department, to take up at once the research problems of his chosen field without having to acquire for the first time a knowledge of tools and methods in the use of which he already should have acquired skill.

ROBERT E. CHADDOCK

Columbia University

EUGENICS CONGRESS

The Second International Congress of Eugenics which recently has completed its sessions at the American Museum of Natural History, had a far larger and more enthusiastic attendance than was foreseen. The congress was planned with certain major policies in view. The first of these involved the presentation to those attending the congress of a large body of biological facts which might serve as a firm foundation upon which any theoretical treatment of eugenics should rest. Scientific communications dealing with human and comparative heredity formed, therefore, the first section of the congress. There were thirty-two such papers on the program. Among those contributing papers to the program were Dr. E. G. Conklin, Dr. C. E. McClung, Dr. C. R. Stockard, Dr. A. J. Rosanoff, Dr. H. S. Jennings, and Dr. Lucien Cuénot of Nancy, France. The latter gave the leading address of this section.

A dinner for all those interested in genetics was held during the period of the congress and was attended by about fifty members. This gave opportunity for an evening of pleasant social intercourse and for the exchange of ideas. The

splendid backing which the geneticists gave the congress formed one of its most striking features and was the greatest single factor which contributed to its success, inasmuch as it removed completely the criticism formerly directed against eugenics to the effect that as a science it depended too much on theoretical and too little on experimental facts.

Inasmuch as another criticism, which in the past has made a good deal of trouble for sincere eugenicists, has been the failure of certain agitators masquerading under the title of eugenicists to subscribe to the laws of marriage common to our present social system, it seemed desirable to feature the value and applicability of the human family in eugenic research. Therefore the second section of the congress dealt with this topic. Dr. Lucien March, chief statistician of the French Government, gave the leading address in this section. The twenty-two papers which made up its program included addresses by Dr. Helen Dean King, Dr. David Starr Jordan, Dr. Corrado Gini, Dr. Frederick Adams Woods, and Dr. Roswell H. Johnson.

The importance of the biology of races to the world in general, and to the United States in particular, is evident. The hybrid nature of our population for generations to come is certain and it is, therefore, incumbent upon us to learn all that we possibly can concerning the problems of anthropology which deal especially with racial crossing and combination of racial types. The third section of the congress, therefore, dealt with this topic. The leading address was given by Dr. G. V. de Lapouge of Poitiers, France, who dealt with the topic of race among mixed populations. Among the fifteen papers given were addresses by Dr. Jon Alfred Mjöen, Dr. Frederick L. Hoffman, Dr. Radcliffe N. Salaman, Dr. Ales Hrdlicka, Dr. C. E. Seashore, and Dr. W. McDougall.

The fourth section of the congress dealt with the application of eugenics to the state. Here it was necessary to offer freedom of discussion without allowing extreme radicals and faddists to become rampant. In this respect this section offered one of the greatest problems of the congress. Its success was shown in the large and enthusiastic audience which attended all its sessions, and by the successful dinner which it held during the congress in conjunction with section two. The topics discussed in its thirty-five papers were many and diverse. Throughout, a note of seriousness and regard for the truth, unfortunately absent in some of the eugenic communications of past years, was apparent. Among those contributing papers were Dr. Raymond Pearl, Dr. E. M. East, Dr. D. F. Ramos, Dr. Harriette A. Dilla, Dr. W. McDougall, Mr. Otto E. Koegel, Dr. Stewart Paton, and Dr. V. Giuffrida-Ruggeri.

A large exhibit of the results of a great variety of research in eugenics was held in conjunction with the congress. Dr. H. H. Laughlin of the Eugenics Record Office, Cold Spring Harbor, N. Y., had charge of the assembling and preparation of these exhibits. As a feature for public education and for arousing interest the exhibit will long be remembered by all those who saw it. It remained open to the public for approximately three weeks following the congress. During that time almost one thousand people registered, and the attendance must have been several times this number.

The press notices of the congress and the general treatment by the public were

in marked contrast with those accorded past efforts in the field of eugenics. This is, perhaps, the most striking and encouraging feature of all. It represents an awakened conscience on the part of the American reading public to the fact that racial features determine the course of history, and that their control is far more practicable than was formerly supposed.

It is likely that in the future we shall look back upon this congress as simply a tiny step in the direction of what may then be accomplished, but at the present time it may be said to represent a changed viewpoint and an awakened interest which augurs well for progress in research and in the resulting accumulation of constructive knowledge in this field.

C. C. LITTLE

FREDERICK S. CRUM

Frederick S. Crum, born at West Candor, New York, on December 13, 1872, died accidentally by drowning near Oakland, Maine, on September 2, 1921, in his fiftieth year.

After graduating from Cornell University in 1893 Dr. Crum continued his studies there with statistics as his main field of work, receiving a master's degree in 1895 and a doctor's degree in 1897. His thesis for his master's degree was on the Vital Statistics of Massachusetts, 1850–90. This was in a real sense a preparation for the work of his later life and ripened into his earliest printed articles, one on the Marriage Rate in Massachusetts, which appeared in the QUARTERLY PUBLICATIONS in 1895 (Vol. IV, pp. 322–39) and another on the Birth Rate in Massachusetts, which appeared in the *Quarterly Journal of Economics* in 1897. His thesis for his doctor's degree on The Statistical Work of Süssmilch, which also appeared in this journal, though less an expression of his primary and predominant interest, is one of the few significant American articles on the history of statistics, and is still the one authority on its topic. It is freely cited by the Germans as the fullest summary of the work of their great statistician.

Almost immediately after ending his academic training he entered the service of the Prudential Life Insurance Company and soon became assistant statistician under Dr. Hoffman, a position which he retained with steadily growing usefulness to the company and the country until his untimely death.

Dr. Crum's most important statistical work lay in the fields of infant mortality and vitality, accident statistics, and the decrease in the native American stock. In the field of infant mortality he published a series of investigations (the first appearing in 1913) into the most fatal diseases of infancy and childhood, namely, measles, whooping-cough, diphtheria, and scarlet fever. Here also belong his two studies of infant mortality in the United States during the war years, 1917 and 1918, one on seasonal and geographical variations in infant mortality, and the other on infant mortality and its effect on the after-life-time of survivors. With this group should be placed the standard anthropometric tables for children 6–48 months old, started in connection with the better babies' campaign in his home city of Newark, later extended at the invitation of the American Medical Association, and finally used throughout the country by the Federal Children's Bureau. The campaign to secure periodical measurement of height and weight

of young children and a comparison of the results with these standards has probably been as potent as any single agency in diminishing the mortality and improving the physique of young children.

In the field of accident statistics his published work began about twelve years ago on the invitation of the Department of Commerce and Labor, when he prepared an extended report on accidents to railroad employees in New Jersey, 1888–1907. This was followed by a pioneer paper printed in this journal on Street Traffic Accidents (Vol. XIII, pp. 473–528), which greatly helped to start the present nation-wide movement for accident prevention. In the same field were several papers on the statistics of boiler explosions, on automobile fatalities, and on public accidents and their cost, all prepared, I believe, as by-products of his work as chairman of the Statistical Committee of the Public Safety Section of the National Safety Council. That Committee prepared standard forms to be used by city police departments in reporting public accidents. Before this was done some large cities made no reports and the others lacked a uniform system of reporting, so that no general results and no comparisons in space or time were possible. These forms for accident reporting are being put into general use for the first time this year. Their introduction is perhaps the most important recent contribution toward the prevention of accidents.

No American student has done more significant work in ascertaining the facts about "race suicide." After a preliminary article in a popular monthly he published in this journal in 1914 (Vol. IV, pp. 215–22) a statistical study of genealogical results, under the title "Decadence of the Native American Stock," which elicited a personal note of appreciation and congratulation from Mr. Roosevelt.

These diverse but related lines of effort were all unified and inspired by an intelligent and persistent aim to prepare the way for social improvement through an accurate determination of the facts.

Dr. Crum leaves a widow and two children.

WALTER F. WILLCOX

Cornell University

CORRESPONDENCE

LONDON, November 2, 1921

To the Editor of the
Quarterly Publications of the American Statistical Association:

Sir:

Dr. Wm. H. Davis, chief statistician for vital statistics of the Bureau of the Census, Washington, draws my attention to my statement on page 820 of the September issue of your Quarterly Publications to the effect that "it would probably be impracticable for the federal government. . . . to arrange for a federal as well as a state analysis of every single death in the country," and informs me that this is being carried out: "all the transcripts of death certificates are edited in a uniform way so that the statistics from all parts of the registration area are strictly comparable."

I shall be obliged if you will publish this correction on the point of fact.

I am, Sir,

Yours obediently,

ARTHUR NEWSHOLME

MISCELLANEOUS NOTES

Public Health Institute. Preceding its Fiftieth Annual Meeting, which was held in New York City, November 14–18, the American Public Health Association conducted a Health Institute from November 8 to 11. Practical demonstrations were arranged for visiting delegates, affording to public health workers an opportunity to see the actual operation of established methods applicable to various phases of health work, the hygiene of mother and child, public health nursing, social work, sanitary engineering, laboratory procedure, control of communicable diseases, mental hygiene, industrial hygiene, and vital statistics. The group interested in vital statistics visited the Metropolitan Life Insurance Company; the office of the Secretary of the 1920 Census Committee for New York City; the Division of Vital Statistics of the Health Department; a central exhibit of Record Forms, applicable to various phases of health work, at the National Health Council; and the rooms of the New York Tuberculosis Association. The group was shown the handling of vital facts both by hand methods and by mechanical devices; the districting of a city for purposes of administration and for research in reference to health problems; the actual operation of a municipal division of vital statistics; and the methods of utilizing data with reference to a particular disease such as tuberculosis.

New Advisory Committee on Statistics. The National Child Health Council plans to organize a number of advisory committees to collect, study, and make available information on subjects that contribute to healthful childhood. For the guidance of its member organizations and others interested in promoting the health of children, it is now organizing an advisory committee on statistics. This committee will have the immediate task of supervision, from the statistical standpoint, of the preliminary studies and the definite plans for the child health demonstration to be conducted in Mansfield and Richland County, Ohio. Its present membership includes Joseph A. Hill, representing the U. S. Bureau of the Census; Robert M. Woodbury, representing the U. S. Children's Bureau; Edgar Sydenstricker, representing the U. S. Public Health Service; and the following members at large: Truman L. Kelley, Louis I. Dublin, W. J. Lauck, William F. Ogburn, Emma Duke, Bird T. Baldwin, and Robert E. Chaddock. The American Statistical Association has still to appoint three representatives.

Report on Official Statistics of the United Kingdom. A Report on the Collection and Presentation of Official Statistics, prepared by a committee appointed by the Cabinet of the United Kingdom, was recently issued. The committee was appointed in response to a petition presented by a committee of the Royal Statistical Society and signed by numerous other persons interested in statistics, and was composed of various departmental officers. It recommended the establishment of a permanent consultative committee for the coördination of official statistics, the members to be nominated by the different departments. Powers and procedure of the committee were defined in general terms. It was also recommended that a comprehensive index should be prepared to the numerous

volumes of a statistical character or containing statistics which are published by the various government departments.

Bulletins of the Canadian Census. The Dominion Bureau of Statistics, Canada, has begun the issue of bulletins giving the preliminary results of the Decennial Census of Population and Agriculture, taken as of date June 1, 1921. The bulletins issued to date have covered the population of some eighty of the larger cities and towns.

Annual Meeting of the Statistical Association. The Eighty-third Annual Meeting of the Association will be held in Pittsburgh, Pa., on December 27–29 inclusive. Programs of the meetings will be mailed to the members of the Association, together with the necessary information regarding trains and hotels.

M. Lucien March, Directeur honoraire de la statistique générale de la France, visited the United States as a delegate to the International Eugenics Congress. M. March returned to France in November after remaining in this country during September and October.

Mr. G. H. Knibbs, formerly Commonwealth Statistician of Australia, has been appointed chairman of the Commonwealth Institute of Science and Industry.

Mr. R. von Huhn, formerly with the Edison Storage Battery Company, has been appointed special agent for the government and is in charge of the research statistics in the Department of Commerce in Washington.

Mr. Arthur J. Hirsch has been appointed chief clerk of the Bureau of the Census. Mr. Hirsch was chief of the Division of Agriculture in the same bureau.

Mr. Ethelbert Stewart has been serving as commissioner of labor statistics since the resignation of Dr. Royal Meeker. His appointment has been confirmed by the Senate.

Mr. L. R. Gottlieb, editor of the *Bankers Economic Service*, is at present engaged, as financial expert for the United States Treasury Department, in a study of Germany and France in connection with the Conference on Limitation of Armament.

NEW MEMBERS ELECTED SINCE SEPTEMBER 1, 1921

Ackermann, Carl, English Dept., Capitol University, Columbus, Ohio
Baldwin, Alden W., Gilbert and Barker Mfg. Co., Springfield, Mass.
Belcher, Earl J., American Tel. and Tel. Co., 195 Broadway, New York
Bradford, Donald S., Boston Store, Milwaukee, Wis.
Craddock, Catherine E., Jones and Baker, 50 Broad St., New York
Goodman, H. C., Daily News Record, 8 E. 13th St., New York

Laning, F. H., The White Company, Cleveland, Ohio
Moffett, Clemens, 13 Thomas St., New York
Monroe, Walter S., Bureau of Educational Research, University of Illinois, Urbana, Ill.
Norlie, O. M., Dept. of Psychology, Luther College, Decorah, Iowa
Phillips, Frank M., George Washington University, Washington, D. C.
Riley, Alfred J., Mutual Benefit Life Ins. Co., Newark, N. J.
Rubin, Benjamin, 6448 S. Halstead St., Chicago, Ill.
Russel, Albert Wells, Russel Wheel and Foundry Co., Detroit, Mich.
Russell, Samuel W., Commercial Research Dept., Swift and Co., Chicago, Ill.
Webb, Walter L., Utah-Idaho Sugar Co., 310 Vermont Bldg., Salt Lake City, Utah

REVIEWS

Supplement to the Seventy-fifth Annual Report of the Registrar-General of Births, Deaths, and Marriages, in England and Wales. Part II. Abridged Life Tables. London. 1920. Pp. xlvi–65.

The subject of this supplement is the preparation of abridged life tables. Short methods are explained and illustrated, showing how to calculate the various functions of a life table to a degree of accuracy closely approximating the values obtained by the more elaborate and extended methods accepted by actuaries. It is stated that "an abridged life table of sufficient accuracy for the purposes of a Medical Officer of Health may be constructed in three or four hours, provided ordinary death-rates in various age-groups of the population concerned are available." Numerous contributions on this subject have been published in statistical and actuarial journals, but this is the first official government publication in English of which I am aware that is devoted entirely to the subject of computing abridged life tables.

In Part I of the supplement George King gives a very satisfactory solution of this problem and illustrates his theory by a concrete example. His method or obvious extensions thereof would seem to be sufficient to meet the requirements of those whose mathematical attainments are limited but who nevertheless would like to prepare abridged life tables. King's method is one whereby the abridged life table values, like those in the extended table, are calculated from the original statistics without reference to the values in any other table.

The method explained in Part II of the supplement is interesting because it proceeds along entirely different lines. It is presented by E. C. Snow, M.A., D.Sc., and appears on pages v to xxv of the supplement under the title, "An Elementary Rapid Method of Constructing an Abridged Life Table." By an abridged life table is understood a life table in which the values of the mortality functions are not given for every single year of age, but only for a sufficient number of age intervals to serve the practical requirements of registrars and of health officers. In the present supplement the age intervals for the abridged life tables are 0–1, 1–2, 2–5, 5–10, 10–15, 15–20, 20–25, 25–35, 35–45, 45–55, 55–65, 65–75, 75–85, 85–95, 95–. The underlying assumption in Dr. Snow's method is that the probability that a person aged x will live n years, ${}_np_x$, can be expressed as a linear or quadratic function of the observed death rate, r. This relation or equation varies, not with the age group, but with different sections of the usual range of values covered by the death-rate, r. These equations are obtained by the solution, by the method of least squares, of a series of equations of the type

$$p = a + br,\ p = c + dr + er^2.$$

The values of r are observed death rates, and the corresponding values of p (or ${}_np_x$) are taken from certain arbitrarily selected life tables already calculated by some extended method. For example, when $x > 10$ and $.00300 \leqq r \geqq .00370$, the

quadratic relation for five-year groups is

$$_5p_x = .98152 + 5095.5(.00383 - r)^2,$$

and when $.00500 \leqq r \leq .00800$, the quadratic relation for ten-year groups is

$$_{10}p_x = .07286 + 26.8859(.18575 - r)^2.$$

The proof offered for the validity of these assumptions other than that they seem reasonable is the close agreement between the values obtained by the application of the method to those actually calculated by the extended method. The above equations were derived from the following data.

England and Wales, 1910–12

Five-year period			Ten-year period		
Experience of age group	Observed death-rate	$_5p_x$	Experience of age group	Observed death-rate	$_{10}p_x$
15–20 M	.00288	.9861	25–35 M	.00480	.9531
20–25 F	.00314	.9845	30–40 F	.00514	.9498
5–10 F	.00318	.9835	30–40 M	.00614	.9403
5–10 M	.00320	.9832	35–45 F	.00652	.9365
25–30 F	.00367	.9819	35–45 M	.00799	.9224
20–25 M	.00372	.9814	40–50 F	.00838	.9186

It is evident that the observed death-rates are taken indiscriminately from male and female experiences and from different five- and ten-year age groups. The one necessary point is that r in the five-year group must lie about in the range (.00300 – .00370) and in the ten-year group about in the range (.00500 – .00800). This was found to be necessary because in different experiences wide variations in age x were found to correspond to the same value of r. Therefore it would be quite impossible to confine an observed death-rate to a fixed value of x, although n may be fixed as 5, 10, or any other desired term.

Having found $_np_x$ by this method for the age intervals 10–15, 15–20, 20–25, 25–35, . . . 75–85, it remains only to select a radix, say 10,000, at age 10, and to construct the l_x column for ages 10, 15, 20, 25, 25, . . . 85. For the end of the table $\log {}_{10}p_{85}$ is assumed to be a linear function of the observed deat-hrate for the age interval 85 and upwards, and the resulting equation is

$$\log {}_{10}p_{85} = .188106 - 5.67829r,$$

which makes it possible to calculate l_{95}.

In order to calculate the functions L_x, T_x, in the stationary life table population, and the complete expectation of life, $\mathring{e}_x$, it is assumed that

$$l_x \cdot {}_nk_x = l_x + l_{x+1} + \quad . \quad . \quad . \quad + l_{x+n-1},$$

and that $_nk_x$ can be expressed as a linear or quadratic function of $_np_x$, or what amounts to the same thing, as a linear or quadratic function of r. The equation obtained for the five-year group is

$$k = 3.0914 + 1.9084p.$$

As in the case of expressing p in terms of r by a series of equations depending on the value of r, so a series of equations was determined for ${}_{10}k_x$ depending on the range of p. For example, when $.55 \leq {}_{10}p_x \leq .80$, then

$$ {}_{10}k_x = 14.4520 - .87293(3.2566 - p)^2. $$

For $x = 95$, ${}_{10}k_{95}$ was arbitrarily taken equal to 3. Having found the values of

$$ l_{10} \cdot {}_5k_{10} = l_{10} + l_{11} + l_{12} + l_{13} + l_{14}, $$
$$ l_{15} \cdot {}_5k_{15} = l_{15} + l_{16} + l_{17} + l_{18} + l_{19}, \text{ etc.}, $$

to the end of the table, the values of L_{10}, L_{15}, etc., and T_{10}, T_{15}, etc., are easily calculated. Then the complete expectation of life is derived from the relation $\mathring{e}_x = T_x/l_x$. The values found in this way for ages 10 and upwards are very close to the values in the extended tables and may be regarded as entirely satisfactory for the purposes of the registrar and health officer.

The treatment for ages under 10 is somewhat different. In fact, Dr. Snow divides the discussion into two main parts, ages under 10 corresponding approximately to falling or decreasing values of the death-rate r, and ages over 10 corresponding to rising or increasing values of the death-rate r. The treatment for ages under 10 is divided into four parts corresponding to the age intervals 5–10, 2–5, 1–2, 0–1. Linear equations are found for the 5–10 and 2–5 intervals connecting ${}_5p_5$ and ${}_3p_2$ respectively with r. Similarly for ${}_5k_5$, ${}_3k_2$, and p. For the interval 1–2 the relation between p_1 and r is hyperbolic:

$$ p_1 = .07434 + .92488 \left(\frac{2-r}{2+r} \right); $$

in other words, a linear relation is established between the chance of living a year and the observed central death-rate.

For the first year of life, age interval 0–1, p_0 is taken equal to the complement of the infantile mortality per birth, while

$$ k = \begin{cases} .7646 + .2354 \left(\frac{l_1}{l_0} \right) \text{ for males,} \\ .7871 + .2129 \left(\frac{l_1}{l_0} \right) \text{ for females.} \end{cases} $$

The proposed method is not novel but is rather a natural attempt to express mortality functions for arbitrary age intervals in terms of the observed death-rates by means of simple mathematical relations, namely, linear, quadratic, one to one hyperbolic. It is a suggestive line of thought, and a careful study of the report will well repay the student of vital statistics. To the actuary and mathematician it suggests further developments in the study of analytical relationships between the observed death-rate and the several functions appearing in the life table. It is not at all unlikely that these relations may be quite fully determined and placed on a more satisfactory mathematical foundation, as far as the theory and the proofs are concerned, than is indicated in this supplement. It is desirable to know not only the mathematical relation or equation connecting observed and computed functions, but also to be able to indicate the degree of approximation to which the computed function is obtained.

The explanation of the method is followed by ten pages of specially prepared numerical tables to facilitate the calculation of p from r and k from p, and a table of proportional parts. The remainder of the report includes sixty-five pages of tables, for males and for females, as follows: mean annual death-rates in several age periods in the areas for which life tables are given, 1911 and 1912; l_x, survivors at several ages per 100,000 born; $l_x - l_{x+n}$, deaths in several age periods per 100,000 born; $\mathring{e}_x$, complete expectation of life at several ages; survivors in the several areas as percentages of the survivors at the same age in England and Wales; expectations of life in the several areas as percentages of the expectations at the same age in England and Wales. The tables are presented on the whole in good form, except that in some cases, for example, on pages 31 and 63, the entire page is closely packed with figures without horizontal spaces every fifth or tenth row to relieve the eye and assist in following across the page. Such forbidding arrays of figures are not infrequent in the publications of this office, and the practice is much to be regretted.

JAMES W. GLOVER

University of Michigan

Foreign Exchange Before, During, and After the War. By T. E. Gregory, Cassel Reader in Commerce, University of London. [Oxford University Press. 1921. Pp. 116.

The chief purpose of this excellently written little volume is evidently to restate in brief compass and in simple language the theory now associated with the name of its leading modern exponent, Professor Gustav Cassel. Little attempt is made at inductive verification. The statistical aspect of the problem, though referred to summarily in Chapter VI, "The Present Position of the Foreign Exchanges," is mostly relegated to a brief appendix of tables of exchange rates, prices, and paper money in the leading countries, which are not specifically related to the text.

As a restatement of the Cassel theory Mr. Gregory's essay accomplishes its purpose well. The first two chapters, on the exchange market and its meaning, and how to read the exchange article, are introductory, giving only enough of the technique to make the succeeding discussion of theory intelligible to the uninformed reader. The body of the book is in Chapters III to VI, where the writer discusses, in very broad outline only, the methods and forces governing exchange fluctuations under the conditions of a gold standard as before the war, the devices adopted and suggested for stabilizing the exchanges during the war, and the factors which govern the movements of "dislocated exchange" such as Europe has been experiencing. Following Cassel, he distinguishes between daily or short-time fluctuations which are but a reflection of market changes in the supply of and demand for bills, and the underlying movement of rates, which with Cassel he holds to be fundamentally determined by relative price levels and price changes in the trading countries.

Mr. Gregory insists upon dividing exchange experts into two camps, those who, like himself, think that the exchanges are dislocated *solely* by price inflation,

and those who think that they are dislocated *solely* by reason of adverse international balances. The latter he thinks a superficial view, even when by adverse balance is meant the balance of payments as distinguished from the mere merchandise balance, because behind the changes in the international balance which affect exchange rates there are price changes; "the adverse balance is an effect and not a cause." The general conclusion is set forth in Cassel's own terms: "If we know the increase in the price levels in two countries, then the actual rates of exchange are at parity when the ratio between them corresponds to the purchasing power parity as expressed by the ratio of the price levels in the two countries. The peculiarity of the purchasing power parity is that it is a moving par; it varies with every change in the price levels in the two countries." If relative price levels, short-time fluctuations apart, do not conform to the "purchasing power parity," the currencies are said to be "undervalued" or "overvalued," as the case may be, in terms of the purchasing power parity; and it is a cardinal point of the Cassel doctrine that this situation sets at work forces which by effecting changes in international balances will tend to restore the exchanges to equilibrium at the purchasing power parity.

The writer alludes to the failure of the price and exchange data thus far to bear out the requirements of the theory, and cites the assertion of the Dutch economist, Miss E. C. Van Dorp, that Cassel's solution of these "abnormal deviations from the purchasing power parity" as being due to "one-sided restrictions" on trade has laid him open to the charge of being an apostate from his own theories by lugging in the adverse balance explanation by the back door. Mr. Gregory is himself inclined to ascribe this disconcerting failure of the theory to work, thus far, to the defects of price indices and to speculative influences.

It is not possible in a brief review to consider adequately the merits and the limitations of the Cassel theory. The doctrine apparently contains the implication, so often repeated despite frequent refutation and the easily observed facts, that international price levels tend toward an equality, serious departure from which brings about, in the absence of gold movements, a condition of depreciated exchange.* In Mr. Gregory's words, under normal conditions "there will tend to be established a uniform price level, or what is the same thing, the value of money in the different places will tend to be the same." To one familiar with the workings of the balance of international payments and the theory of reciprocal demand, this fallacy does not require formal disproof. The fact that before the war prices were, and had for generations been, higher in the United States than in England, higher in England than in Germany, higher in Germany than in Italy, higher in Italy than in China, is itself a sufficient refutation.

The assertion of a normal tendency toward international price uniformity, however, is not in my opinion necessary for the maintenance of the Cassel doctrine. My own principal difficulty with the theory arises from its insistence upon comparative price changes as the first and only fundamental cause of exchange depreciation. I cannot agree that "the adverse balance" is always

* The same assumption underlies Nicholson's distinction between "general" and "specific" depreciation; see *Inflation*, pp. 69–71. For criticism, see my article on "Foreign Exchange, Prices, and the Course of International Trade," *The Annals*, May, 1920.

"effect and not cause," or that price changes are always cause and not effect. As I have several times previously stated, I see no reason why students of foreign exchange theory should divide into the two camps Mr. Gregory designates. There is truth in both views. Adverse changes in the balance of payments and adverse changes in international price levels, due, for example, to "overissue" of inconvertible paper money, interact upon each other. Either may in a given instance be the cause of the other. Either, if sufficiently pronounced and prolonged, is capable of bringing about a condition of depreciated exchange; both together are bound to. Occasionally we find instances in which these two fundamental determinants of exchange rates counteract each other for a time, as when a country is borrowing capital from abroad at the same time that it is issuing more inconvertible paper. That was clearly the case in Argentina in the years preceding the Baring Panic. It was also the case during the period of our participation in the World War, when our government was advancing billions of dollars of credits to European governments at the same time that they were multiplying their issues of paper money. In this instance the loans were *consciously* applied toward exchange stabilization, but the principle was precisely the same. Ultimately, to be sure, both of the changes here instanced, in the international balance and in the volume of money, would unite to produce exchange depreciation, for the new loans would create large interest charges payable by the borrowing nation. But to say that either the price factor or the balance-of-payments factor is the fundamental determinant of the underlying exchange movement, in the sense that the one factor exercises an independent control over the other and is not in turn influenced by it, appears to me to require more proof than has yet been adduced. My own investigations thus far support the contrary view. It may be pointed out, too, that in a sense neither factor is fundamental or ultimate; both are but different forms of borrowing. An issue of inconvertible paper money is in the nature of a forced internal loan; an adverse international balance is evidence of international borrowing. The only "fundamental" determinant of dislocated exchange in Europe in recent years is the war itself, which gave rise to both forms of borrowing on an unprecedented scale.

Concerning the failure of the price and exchange data to bear out the requirements of the Cassel theory, to which allusion has already been made, it is interesting to note a striking instance within the last twelve months. During the war the Argentine peso was at a high premium in terms of both the dollar and the pound. Last fall and winter the prices of the exportable products of Argentina fell 50 to 70 per cent, and they have since remained decidedly weak. Deflation has been more rapid there than either here or in Europe. On the Cassel reasoning, the peso should therefore have appreciated in value relative to other currencies this past year; but in fact it has fallen markedly in terms of both the dollar and the pound. The dollar has gone to a premium of 20 to 30 per cent, and the pound after having been markedly and continuously depreciated since 1916 reached par in Buenos Aires last February.

As Mr. Gregory points out, "there is nothing essentially new" in the Cassel doctrine. "From the days of the Bullion Committee of 1810 it has been part of the accepted body of economic teachings that the increase of inconvertible

paper money would turn the exchanges against the country." The theory associated with the name of Cassel "is nothing but an elaboration" of this fact.

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Sugar Supply and Prices. Report of the Federal Trade Commission, November 15, 1920. Government Printing Office. 1920. Pp. 205.

It is unfortunate that this report of the Federal Trade Commission was not published a few months earlier when public interest in the sugar situation was at its peak, and responsibility for the abnormally high prices and shortage of 1919–20 was laid, in turn, upon President Wilson, because in his solicitude for the League of Nations he had neglected the humbler task of authorizing the Sugar Equalization Board to purchase the Cuban crop of 1919–20; upon Professor F. W. Taussig for his minority report to President Wilson, advising against government control of the sugar business for 1920; upon Attorney-General Palmer for authorizing the maximum price of 17 cents for Louisiana sugar, which action, it was alleged, had excited the "cupidity of Cuban producers"; and upon other individuals and classes of humbler order and significance. It will be recalled that at this time even the Lusk Committee turned from profound research into the machinations of the "Reds" to investigation of the sugar situation.

By House Resolution of October 1, 1919, the Federal Trade Commission was directed—

> to make an *immediate* investigation and report to the House of Representatives its findings on the subject of sugar used for domestic and manufacturing purposes, both as to its market price, wholesale and retail, and whether these prices are reasonable; and the amount of sugar now in the United States, both raw and refined, in the possession of sugar refiners, wholesale dealers, or in storage; said investigation to involve a report of supplies in the possessions of the United States, raw and refined, and the causes of the present sugar *scarcity*.

Here economists and statisticians have the startling exhibit that the much maligned "long run tendency" in economic life, though grinding slowly, grinds more rapidly than the statistical or economic investigator. For by the time the Commission was enabled to issue its report* in response to the above resolution, the record-breaking high price of sugar in the United States had attracted imports from all parts of the globe, and with supplies far in excess of demand the sugar market was in a state demanding an investigation into the causes of the sugar "glut," not scarcity, and a study of the technique of the moratorium, such as had been declared in Cuba, and other devices to prevent wholesale ruin to sugar producers, refiners, and dealers could logically have been substituted for the inquiry into the reasonableness of prices.

Although the original function of the investigation was not, therefore, fulfilled, the Commission has rendered a valuable service in thus compiling from many sources and arranging in compact and logical form all the significant economic and statistical data bearing on conditions in the production and distribution of sugar during the summer of 1919 and the first part of 1920, a period of unusual

* Dated November 15, 1920.

interest because it marks the transition from government control of the sugar trade to ordinary competitive conditions, a rapid shift from stabilization to inflation and then deflation.

In the first chapter there is an admirable section giving accurate definitions of the statistical units hitherto employed carelessly in the trade and in government publications. Statisticians and all others who have had or will have occasion to deal with sugar statistics will be grateful for these few pages. In the second chapter there is presented a careful and ably interpreted statistical and economic history of supply and demand conditions in 1919 and the first part of 1920, explaining "inter alia" the various factors leading to the shortage of 1919, the measures taken by the Sugar Board and the Food Administration to relieve the shortage, the activities of speculators and the development of that rather curious practice of "refining on toll" whereby refiners protected themselves against rapid price fluctuations in raw sugar. Chapter III treats of the prices prevailing in the period under consideration for nearly all branches of the sugar trade: the beet sugar producers, the Louisiana cane producers, the refiners, and the distributors. "Evidence on the reasonableness of sugar prices, in so far as costs, investments, and profits are determining factors" was not presented because sufficient data had not been secured on these matters by the Commission. Chapter IV gives the history of the policy of the government in relation to the 1919–20 Cuban crop, which has become generally familiar through considerable discussion in the press. A series of exhibits is appended, including the contracts between producers and various government agencies, statistical statements, etc.

The statistical material embodied in the report is derived in general from three sources, the Statistical Department of the United States Sugar Equalization Board, the sugar trade journals, and the investigations of stocks, sales, and purchases of distributors, conducted by the Commission itself. By combining the last-mentioned material with the published sugar statistics of the Sugar Equalization Board, the data in the Food Surveys of the Department of Agriculture, and the cost studies of sugar production and refining of the Tariff Commission, the investigator will find a statistical record of the operations of one large industry unique in the completeness of the material available for the period covered.

The Commission has drawn some definite conclusions from the investigation made. It has found that "the price of sugar was kept comparatively low during the war and throughout 1919 for most of the 1918–19 crop." Secondly, it found that "while there is no doubt that expressions of public opinion were generally opposed to such a policy, the facts as since revealed present a strong argument that the continuation of government control, at least until more stable trade conditions were reached, would have been helpful." It recommends, therefore, that "at least portions of the food-control act be enacted into permanent law for the purpose of preventing unreasonable storing, acquiring, accumulating, and hoarding of food, fuel, and other necessaries of life." As a means for making these provisions of the food-control act effective, the Commission recommends that power be given some government agency to obtain current information on production, stocks, costs, etc.

It is not for the humble reviewer to discuss the wisdom or soundness of these

conclusions and recommendations. A word may be said, however, on the conclusion that "continuation of government control . . . would have been helpful." If the Cuban 1919–20 crop had been purchased by a government agency such as the Sugar Board at 6½ cents, ultimate return to normal market conditions would unquestionably have been delayed for another year and Cuban raw sugar would not now be selling at slightly over two cents per pound f.o.b. Cuba. At this late date it seems reasonably clear that it was just as well that we took our medicine in 1919 and now have the readjustment over with, once for all, although there was no necessity for much of the "economic friction" suffered in the process. What was required was not another period of government control but the early formulation of a definite government policy in 1919 as to the mechanism of the control. Under the conditions prevailing in 1919 the Executive was so overwhelmed with grave responsibilities that he could not give sufficient attention to the sugar situation. As for Congress, the slow process of national legislation with its hearings and "filibusters" could not meet the requirements of a business situation. Consequently the sugar trade was kept in a state of suspense for over six months. The experience with government control of sugar has thus given added proof that simplification and reorganization of the machinery of government must precede any increase in its sphere of activity.

There is warrant for justifiable criticism of the report for the inclusion of much material, especially among the exhibits, that is already available in other government documents. Thus, for example, the war-time contracts between sugar producers and the government agencies have already been printed in the annual reports of the Food Administration, in Senate and House Hearings on Sugar, in the sugar trade journals, etc. Such duplication of printing should be avoided wherever possible. A little of the $17,359.80* expanded for this inquiry might have been saved thereby.

JOSHUA BERNHARDT

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An Index of the Physical Volume of Production. By Edmund E. Day. Reprinted from the *Review of Economic Statistics*, September, 1920–January, 1921. Cambridge. 1921. Pp. 70.

Professor Day finds that "economic events of recent years have emphasized the need of an index of the physical volume of production." These events are the rise of prices since 1897, the lack of information as to "overproduction" or "underproduction," and the growth of an opinion that only an increase in physical units of goods and services indicates an improvement in economic condition. Consequently, on all accounts, a measure of the social product by units of volume is called for. "Almost no endeavor" has been thus far made to create such an index, and therefore Professor Day undertakes the task.

In attacking the problem before him Professor Day's first work is to pass in review the material available for use. He sketches data relating to the output

* As reported in the House Hearings on the Sunday Civil Appropriation Bill, 1922. Government Printing Office, 1921. P. 512.

of the twelve most important crops of the country, basing his figures on census and the Department of Agriculture statistics; adds mining figures including gold, silver, coal, copper, lead, zinc, petroleum, coke, etc., basing his figures on the Geological Survey, Bureau of the Mint, and Iron and Steel Institute statistics; and closes with manufactures under which he includes fourteen groupings, depending largely on the census for his information.

After presenting his agricultural figures the author prepares an "unadjusted index." To do this he calculates the average of the items of each series for the five years 1909–13 (the base period). Next, he expresses the items of the several series as percentages of the respective base period averages. A system of weights in accord with values is then introduced. A geometric mean of the twelve percentage relatives is then computed. The result is the unadjusted index for agriculture. An adjusted index is derived from the unadjusted by first determining the lines of secular trend and expressing every item of each crop series as a percentage of the corresponding ordinate of the line of the series' secular trend. Finally, there is computed for each year a weighted arithmetic mean of the twelve percentage relatives of the year, the same weights being employed as for the unadjusted index.

In mining an unadjusted index is computed along lines similar to those employed in agriculture. "Adjustment" is then effected by making due allowance for increases or decreases in production which may properly be considered normal. The methods are like those already outlined for agriculture.

The process of deriving the unadjusted index for manufactures is somewhat more complex than in the other two branches of production—agriculture and mining. Comparisons are made for intercensal years with other data drawn from independent sources. In the main the methods do not differ substantially from those used in the earlier sections of the study.

In preparing the adjusted index for manufactures the first step is the determination of the lines of trend for the thirty-three original series chosen. Percentage relatives resulting from the reduction of the items of the original series to percentages of normal are then computed. Finally, the adjusted index is secured by steps exactly paralleling those taken in the computation of the unadjusted index.

With the three main indices in hand, Professor Day confronts the question of combining them into a single figure. To this end there is computed an unadjusted form of index by calculating a weighted geometric mean for each year of the twenty-one year period under study. The weights used correspond to the aggregate values of production in agriculture, mining, and manufacture during the census year 1909. The adjusted index is a weighted arithmetic mean of the three separate indices. Comparison is then instituted with some familiar business indices, and coefficients of correlation are worked out. This brings the study to a close.

Professor Day is entitled to full credit for a careful and systematic piece of work in a field that has not heretofore been much attended to. The objections to any such undertaking, developed on any basis, and its faults as a means of gauging productive activities or economic welfare, are already obvious theoret-

ically, and need no discussion. Certain serious questions of a practical nature are, however, properly raised. Are basic statistics sufficiently refined to justify or require methods as elaborate as those used in Professor Day's data? Is it not probable that the use of an ordinary weighted arithmetic index would produce results as satisfactory? Such consideration seems to transfer the emphasis in the study of production to the matter of obtaining original data instead of allowing too much effort to be spent in elaborate computations. Since production is not as yet upon anything like as exact a statistical basis as prices and foreign exchange, undue refinement is misleading. It is worth noting, too, that while annual figures on such subjects are of historical interest, the real use of such indices as those of Professor Day will appear only when a simple and practical monthly measure of production is obtained. In general, a question must always arise as to how far we can understand and use composite results when we have so little relative knowledge of the different parts of the field. Research into the condition of special industries is an essential supplement and may be considered the next field to be cultivated, rather than the general index field.

H. Parker Willis

War Costs and Their Financing. By Ernest Ludlow Bogart. New York: D. Appleton and Company, 1921. Pp. xxiv, 510.

One of the most remarkable features of the World War was the ability of the various belligerents to endure the economic strain for so long a period. At the outbreak of the conflict it was generally predicted that it would be short because the expensiveness of modern warfare would soon lead to the economic exhaustion of the belligerents. But for four long years the struggle raged with constantly augmenting fury and mounting expenditure. Consequently to every belligerent the problem of finance—the raising of funds to meet the huge expenditures involved—presented itself as one of enormous magnitude.

In *War Costs and Their Financing*, Professor Bogart has calculated the expenditures of the principal nations engaged in the war and has described the manner in which these expenditures were provided for. His study of war financing necessarily involved consideration of paper money issues and bank credit in the various warring countries. Less directly related to the main discussion of the book are chapters dealing with the financing of Europe after the war, and with post-war problems of currency, debt, and taxation. In the final chapter an attempt is made to summarize all the costs of the war, indirect costs as well as the direct outlays of the belligerent nations. An introduction is contributed by R. C. Leffingwell, former assistant secretary of the Treasury. In the appendices are contained various moratorium proclamations, liberty bond acts, and statistics of taxation and debt in the United States.

In this volume Professor Bogart gives only a summary statement of the expenditures of the belligerent countries. More detailed calculations are found in his study on *Direct and Indirect Costs of the Great World War*, from which most of the statistics in the present book are taken. His final figure for the total net

expenditures of all the nations during the four and one-half years of war is $186,000,000,000. This amount is to a considerable extent based upon estimates and guesses because of the failure of some of the belligerent countries to publish statements of expenditures, and of others to distinguish clearly between war and ordinary expenditures.

Throughout the book foreign currencies have been converted into dollars at approximately the pre-war mint par rates of exchange. In order to combine the figures for various countries, it was, of course, necessary to reduce all currencies to a common denominator. Perhaps the method employed by the author was the only practicable one; but it should be realized that owing to the depreciation of the currencies of all of the belligerents as compared with our dollar, the author's method of conversion leads to an exaggeration of the total costs of the war. For instance, assume that Germany's expenditures were 160,000,000,000 marks. Converted at the approximate pre-war rate of 25 cents to the mark, this sum is equivalent to $40,000,000,000. But considering the heavy depreciation of the mark, can it be said that the expenditure of 160,000,000,000 marks represented the same amount of economic goods as that of $40,000,000,000?

The fluctuations in the values of the currencies of the various countries give a tinge of unreality to much of the statistical estimation of the cost of the war, and necessitate wariness on the part of the investigator in drawing conclusions from his statistics. It is to be feared that in some cases the author himself has stumbled. For example, in order to convey some impression of the huge cost of the war, he states that during its last year it was in some countries already costing more in a single year than the estimated pre-war income of the whole people, and in all other countries except the United States it was approaching very close to this point. In support of this statement there is given a table showing in dollars the annual pre-war income of various countries and their war expenditures in 1918. But the war expenditures of 1918, as the author elsewhere clearly notes, were incurred in terms of a currency grossly depreciated as compared with the pre-war currency. In other words, the measuring rod of value in 1918 had become an entirely different unit from that of 1913, and conclusions based upon the assumption that they were identical are unwarranted. Similarly, a distorted impression is created by the tables on pages 371 and 372 in which the pre-war wealth and income of the belligerents are compared with their present debts and interest charges.

If the author was forced in many cases to estimates and guesses in calculating the direct outlays of the belligerent governments, to a far greater degree was he driven to the realm of conjecture in determining the indirect costs of the war, such as those represented by loss of life, property losses, loss of production, loss to neutrals, and war relief, all of which the author estimates at $151,600,000,000. In particular is this true of the item "loss of life," which is placed at approximately $67,000,000,000 for both soldiers and civilians. In view of the widely divergent estimates from various authorities on the value of a human life, these figures might be halved or doubled according to the taste of the compiler. Indeed, it is very questionable whether such an item should be included at all. Stated in terms of human suffering, of heart pangs endured through the loss of

dear ones, the value to be placed upon the lives lost is incalculable. But if the purpose is to show how much worse off the world is economically as a result of the war, as is true in the present instance, the thesis might be defended that the lives lost will ultimately redound to the economic benefit of the survivors because of the lessened pressure of population upon the means of subsistence. In justice to Professor Bogart it should be said that in his previous study on *Direct and Indirect Costs of the Great World War*, from which his estimates are taken, he refers to this capitalization of human lives lost as a "procedure of doubtful statistical propriety."

The bulk of the book is taken up by a consideration of the ways and means employed by the belligerents in providing for war expenditures. In all countries the principal income was obtained from loans, but the policies of Great Britain and of the United States may be clearly differentiated from those of the other belligerents. Only in the two former countries was a serious effort made to finance the war by resort to heavy taxation. The author considers that these two countries fairly fulfilled H. C. Adams' ideal of the financial management of a war, namely, a "tax policy assisted by credits, rather than a credit policy assisted by taxes," although in the United States only about one-third of the total expenditures were provided for by taxes, and in Great Britain only one-fourth. These achievements, however, were so superior to those of any of the other nations that they shine by comparison. In no other important belligerent country were tax revenues sufficient to meet even ordinary civil expenditures and to pay the interest on the debt.

Certain general features in the financial policies of the belligerents may be noted. The usual procedure was to raise funds by the issue of short-term Treasury bills, which were later funded by the issue of long-term bonds. In all countries great reliance was placed upon the banks, which advanced funds to the governments either through the creation of credits, as in Great Britain and the United States, or by the issue of bank notes, as on the Continent. The result of this policy was world-wide inflation of currencies with its concomitants of high prices and increasing governmental expenses. The general employment of excess profits and income taxes is noteworthy. Especially important were these taxes in Great Britain and in the United States. Indirect taxes were relied upon principally by the other belligerents.

The war has left the belligerent nations staggering under colossal debts and saddled with grossly inflated currencies. Professor Bogart thinks that except in the case of the United States there is little probability of any substantial contraction of indebtedness in the near future, unless there is resort to drastic procedure such as capital levies or outright repudiation. He is similarly pessimistic with regard to the likelihood of deflation and price declines. Perhaps he would have been less so had the book gone to press a few months later, in view of the radical fall in prices in most countries of the world during the past year.

In attempting to cover the war financing of all the principal belligerents, the author has had before him a vast field for research. The gaps in the published material available and limitations in space at his disposal have precluded final statistical exactness or fulness of detail in his treatment of the subject. He has,

however, given a very readable general survey of a very large and complex problem. Written in untechnical terms, the general reader especially will find it an enlightening presentation of the financial angle of modern warfare. Furthermore, an estimation of the economic cost of war, although subject to unwarranted use in some specific applications as has been pointed out, nevertheless is worth while because of the force with which the stupendous total strikes the imagination as one of many reasons for the abolition of war.

WILLIAM O. WEYFORTH

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Report on the Pandemic of Influenza, 1918–1919. Ministry of Health of Great Britain. H. M. Stationery Office, 1920. Pp. 577.

Our members will be interested in the material given in this report on the epidemiology of influenza, especially in view of the statement that "the epidemiological features of the pandemic are even more complex and puzzling than the clinical characteristics, and the present report does little more than present a large body of data which await interpretation" (p. xii). American statisticians can join with their British brethren in developing methods of analysis which express the outstanding characteristics of the data. They may help the epidemiologists in arriving at concise descriptions of the epidemic course and allied characters. The epidemiologists are not satisfied with any of the theories so far advanced as explanations of the pandemic. "We have passed, it is true, from the former theories regarding origin—the miasmatic theory, the telluric and climatic theories; the importation theory; or the idea that influenza is a specific disease like anthrax, arising and progressing within narrow and definitely circumscribed limits. These are hypotheses no longer confidently held. They do not answer the fundamental questions raised by the pandemic through which we have passed." Some of the features of the pandemic in its local and general phases which seem to require newer and better statistical measures and aids to interpretation are:

A. DESCRIPTIVE AND ANALYTIC EXPRESSIONS FOR WAVE-FORMS

The data to be handled comparatively are the facts of (1) periodicity, (2) case-incidence, and (3) case-fatality. At present all that can be concluded is that "the broad facts remain that the epidemic presented three waves and that they differed in form. The first, short, sharp and high; the second, slower in formation, flatter, higher and more destructive of life; the third, also slow in formation, lower, more prolonged in decline, and partially reverting to the older influenzal form—a condition of things which suggests a biological factor modified by environment, but not, unhappily, directly controllable by human agency" (p. xiii).

B. MEASURES OF "EXPLOSIVENESS" OR EPIDEMICITY, RAPIDITY OF EXTENSION, DIRECTION OF PANDEMIC COURSE

Professor Raymond Pearl, of the Johns Hopkins University School of Hygiene, in his *Influenza Studies* I, II, III, and IV, has suggested several measures of

explosiveness or epidemicity, and these may be found useful in the analysis of the British data. (*U. S. Public Health Reports*, Aug. 8, 1919, and Feb. 18, 1921.)

C. EXPRESSIONS FOR AGE-TYPE OF INFLUENZA PREVALENCE

Some concise descriptive expressions are needed to establish by "type" the peculiar behavior of influenza in the several age-ranges, as a means of comparison with endemic influenza, or with the pandemics of other years. Such measures would be useful in comparing the several age characteristics of "waves" of the 1918–19 invasion.

D. MEASURES OF CONTINGENCY AND RELATIONSHIP

"Even in a world-wide pandemic the problem of epidemic influenza may be largely an internal problem of each nation, a problem of social relationship, of social factors, of domestic habit and life. The materials of the conflagration came from within." These measures must be adapted, also, to the consideration of the following questions:

1. Are the epidemiological features of the cycle of years within which influenza explodes different from those of the influenza-free cycles?
2. Preceding influenza epidemics, is there a rise in general morbidity of the population, an "epidemic constitution" favorable to influenza? Are there often mild and atypical clinical forerunners of the disease, and parallel or allied clinical maladies? Are influenza pandemics focal points in a system of related phenomena?
3. Are there concurrences, similarities, and inter-relationships between outbreaks of cerebro-spinal fever, poliomyelitis, and outbreaks of influenza, bronchitis, and pneumonia?

E. TESTS FOR THE RELIABILITY OF DATA FOR SMALL GROUPS

Numerous experiments in vaccination against influenza have been made. Some control should be exercised over the data for the smaller groups. Some data are available also for the results of treatment for selected, but small, hospital and domiciled populations. These too must be tested by suitable measures for degree of validity of the crude results.

These are a few of the points of contact between statisticians and epidemiologists with respect to analysis of influenza data. Interest in influenza pathometry seems to have flagged in recent months; the enthusiasm for analysis of influenza data has disappeared. But with the publication of volumes such as the one under review, a renewal of inquiry into the higher statistical characteristics of influenza may follow.

Chapter I, on the history of influenza in Great Britain, will repay reading and thoughtful consideration. Much of the text is devoted to a development of the Sydenham and Huxham observations of influenza in the seventeenth and eighteenth centuries. One feels convinced, after due reflection upon the material in the chapter, that with respect to influenza, we are not entirely through with parts of Sydenham's doctrine of epidemic constitutions.

A closer examination of recent events is given in Chapter II. The "epidemic constitution" theme is taken up. The authors state that "it is prudent not to advance beyond the point . . . that before the influenzas of 1918–19 the quality of diseases affecting the population of this country was abnormal" (p.

36). They quote the opinions of Dr. Hamer and Dr. Crookshank "that the association or juxtaposition of cerebro-spinal fever, polio-encephalomyelitis, the Heine-Medin complex and epidemic influenza is not fortuitous, but may be well founded." Certain observations follow on the epidemic in England and Wales, in Scotland, and in Ireland. A discussion of the forms of epidemic curves (p. 55) contains this statement of interest to statisticians:

> Great improvements in the art of curve fitting and the graphical representation of mathematical formulae expressing biological laws or ways of happening . . . have permitted the hope that by analysis of epidemic curves, light may be thrown upon the genesis of the epidemic. The subject has not, however, yet been sufficiently developed to permit the realization of these hopes in the present connection. Upon the whole . . . the balance of probability inclines to the view that the *materies morbi* itself underwent an evolution similar in form to, but more rapid than, the life cycles of flux and reflux, which, Dr. Brownlee has contended, are principal factors of varying epidemicity. Some light may be thrown upon this by harmonic analysis directed to disclose periodicities of incidence.

At the close of this chapter there is a discussion of the epidemic curves for three British ships. There is a suggestion that if some method of resolving compound epidemic curves into monomodal components were at hand, certain valuable epidemiological observations could have been made.

Chapter VII contains some skillful handling of weather and respiratory disease mortality data. The methodological blunders of students of physiological meteorology in our own country are avoided, and due credit is given to Farr's original work on the subject.

Dr. R. Bruce Low contributes a valuable collection of data on the influenzas in Europe and in the Western Hemisphere in Part II of the volume (pp. 202–348).

Part XII of the Appendix, a note on the periodicity of influenza, by Dr. John Brownlee, concludes the volume. This note expands Dr. Brownlee's short article on the same subject in the London *Lancet* (November 8, 1919). He reiterates his statement that influenza tends to recur in 33 week cycles, as determined by his application of the "periodogram," a devise he borrowed from the physicists. Replying to some criticism of his *Lancet* article, he reinforces his former arguments by a study of the Boston, Massachusetts, figures for the influenzas prevailing between 1889 and 1900. Here he finds a rigid periodicity of 33 weeks.

This collection of British data should be read in connection with Dr. Warren T. Vaughan's "Influenza: An Epidemiology Study," published in the *American Journal of Hygiene*, Monograph Series No. 1, July, 1921.

E. W. KOPF

Official Statistics: What They Contain and How to Use Them. By Arthur L. Bowley, Sc.D., Professor of Statistics in the University of London. Humphrey Milford, Oxford University Press. 1921. Pp. 63.

One's first reaction on examining this little book of sixty-three pages, with less than four hundred words on a typical page, is a sense of disappointment that it is not for trained students of economics and statistics. They would

welcome a book under this title written by the author for them, but they are not the readers the author had in mind when he prepared the present essay. It appears in *The World of Today Series*, and although the purpose of the series and of the book itself is not specifically set forth, they are obviously designed for persons with a definite interest in public questions but with no special training and experience in their study. One who is familiar with the work of our large government statistical offices knows of the almost amazing popular demand for statistics, and the frequent failure to appreciate the almost inevitable limitations to their use. For this type of British citizen Dr. Bowley has written. A similar work designed for American readers would be distinctly worth while.

The book is divided into four chapters, (1) Population, (2) Industry, Trade, and Prices, (3) Income and Wages, and (4) Social Conditions. The writer says: "The main omission is that of reports on revenue taxation and rates; something had to be sacrificed for the sake of space and these reports have a financial rather than a statistical bearing." Under each chapter heading comes several specific subjects; for example, in the chapter on Population are treated, Population, Occupations, Births, Deaths and Marriages, and Migration. The general arrangement under each subject is to discuss first the sources, including briefly the general method of collection and the chief defects and pitfalls. Then, in most cases, is given a selection of figures from the official reports illustrative of their scope, followed by a short textual analysis of the tabulation indicative of how the figures should be used. Throughout run cautions against blind acceptance of figures, and excellent advice is given regarding careful study of how the figures are made before attempting to use them.

LEWIS MERIAM

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